Summary of Second Order ODEs:

(1) $ay'' + by' + cy = 0$

$y = e^{rx} \implies$ characteristic equation $ar^2 + br + c = 0$

(a) Two real unequal roots $r_1 \neq r_2$

$$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$$

(b) ~~Type~~ real equal roots $r_1 = r_2 = r$

$$y = ce^{rx} + c_2 e^{rx} \cancel{x} $$

(c) Complex roots $r = \lambda \pm i\mu$

$$y = c_1 e^{\lambda x} \cos \mu x + c_2 e^{\lambda x} \sin \mu x$$

(2) $ax^2 y'' + bxy' + cy = 0$

$y = x^r \implies$ characteristic equation: $ar(r-1) + br + c = 0$

(a) Two real unequal roots, $r_1 \neq r_2$

$$y = c_1 x^{r_1} + c_2 x^{r_2}$$

(b) one equal roots, $r_1 = r_2 = r$

$$y = c_1 x^r + c_2 x^r \ln x$$

(c) Complex roots, $r = \lambda \pm i\mu$

$$y = c_1 x^\lambda \cos(\mu \ln x) + c_2 x^\lambda \sin(\mu \ln x)$$