

Review List - Things You

should know

Chapter 2 First Order ODE

1) Formula for linear equation

$$y' + p(t)y = g(t)$$

2) Bernoulli Equation $\xrightarrow{\text{reduction}}$ Linear

$$y' + p(t)y = g(t)y^n, \quad v = y^{1-n}$$

3) Separable equations

4) Homogeneous Equation $\xrightarrow{\text{reduction}}$ separable equations

5) Interval of Existence $\left\{ \begin{array}{l} \text{solution} \\ \text{equation} \\ \text{initial condition} \end{array} \right.$

6) Critical points of

$$\frac{dy}{dx} = f(y)$$

and stability / instability of critical points

Chapter 3 Second Order ODE

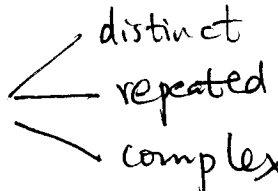
1) Abel's formula for

$$y'' + p(t)y' + q(t)y = 0$$

$$W' = -pW$$

2) Given y_1 , reduction of order to find y_2

$$y_2 = v y_1, \quad v' = \frac{W}{y_1^2}$$

3) Constant coefficients : Three cases 

$$a y'' + b y' + c y = 0$$

4) Method of Undetermined coefficients

$$a y'' + b y' + c y = g(t)$$

5) Method of variation of parameters

$$a y'' + b y' + c y = g(t)$$

$$y_p = u_1(t) y_1(t) + u_2(t) y_2(t)$$

$$\begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix} \begin{pmatrix} u_1' \\ u_2' \end{pmatrix} = \begin{pmatrix} 0 \\ g(t) \end{pmatrix}$$

Chapter 7 Systems of ODEs

1) Abel's Formula for $X' = P(t)X$

$$W' = \text{trace}(P(t))W$$

2) Constant Coefficients $X' = AX$

Three cases $\left\{ \begin{array}{l} \text{distinct} \\ \text{repeated} \\ \text{complex} \end{array} \right.$

3) Method of Diagonalization

$$X' = AX + q(t)$$

4) Method of Undetermined Coefficients

5) Method of Variation of Parameters

6) Fundamental matrix

$$\Psi(t) = (x^{(1)} \quad \dots \quad x^{(n)})$$

$$\Phi(t) = (x^{(1)} \quad \dots \quad x^{(n)}) \text{ such that } \Phi(0) = I$$

Chapter 6 Laplace Transform

$$f(t) \longrightarrow F(s) = \int_0^{+\infty} e^{-st} f(t) dt$$

(1) Important formulas for Laplace Transform

(Table will be given to you)

(2) Solve

$$a y'' + b y' + c y = g(t)$$

with $g(t)$ being

- smooth functions of $e^{\alpha t} \cos \beta t$ $Q(t)$

- piecewise smooth functions

$$\sum_{j=1}^n u_{c_j}(t) f_j(t)$$

- Dirac function: $f(t) \delta(t-c)$

(3) Method of Partial Fractions

Important: $u_c(t) f(t-c) \iff e^{-cs} F(s)$

Chapter 10: Fourier series and method of separation of variables

(1) $f(x)$ periodic with $2L$ period

$$f(x+2L) = f(x)$$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{+\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right)$$

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx, \quad b_n = \frac{1}{L} \int_{-L}^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

(2) $f(x)$ is odd or f satisfies $f(0) = f(L) = 0$

$$f(x) \sim \sum_{n=1}^{+\infty} b_n \sin\left(\frac{n\pi x}{L}\right), \quad b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

(3) $f(x)$ is even or f satisfies $f'(0) = f'(L) = 0$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{+\infty} a_n \cos\left(\frac{n\pi x}{L}\right), \quad a_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

(4) Convergence of Fourier series

$$\frac{a_0}{2} + \sum_{n=1}^{+\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right) = \frac{1}{2}(f(x-) + f(x+))$$

(5) Method of Separation of Variables

- heat equation

$$\begin{cases} u_t = \alpha^2 u_{xx} \\ u(x, 0) = f(x) \\ u(0, t) = u(L, t) = 0 \end{cases}$$

$$u(x, t) = \sum_{n=1}^{+\infty} b_n e^{-\alpha^2 \left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L} x\right)$$

$$b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi}{L} x\right) dx$$

Neumann BC: $u_x(0, t) = u_x(L, t) = 0$

$$u(x, t) = \frac{a_0}{2} + \sum_{n=1}^{+\infty} a_n e^{-\alpha^2 \left(\frac{n\pi}{L}\right)^2 t} \cos\left(\frac{n\pi}{L} x\right)$$

$$a_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi}{L} x\right) dx$$

- Wave equation: $u_{tt} = a^2 u_{xx}$
- Laplace equation: $u_{xx} + u_{yy} = 0$