Review List – Things You should know

Chapter 2 First Order ODE

1) Formula for linear equation

$$y' + p(t)y = g(t)$$

2) Bernoulli Equation $\xrightarrow{\text{reduction}}$ Linear

$$y' + p(t)y = g(t)y^n, \quad v = y^{1-n}$$

3) Separable equations

4) Homogeneous Equation $\xrightarrow{\text{reduction}}$ separable equations

5) Interval of Existence $\leftarrow$ solution, equation, initial condition

6) Critical points of

$$\frac{dy}{dx} = f(y)$$

and stability / instability of critical points

Chapter 3 Second order ODE

1) Abel's formula for

$$y'' + p(t)y' + q(t)y = 0$$

$$W' = -pW$$

2) Given y_1, reduction of order to find y_2

$$y_2 = v y_1, \quad v' = \frac{W}{y_1^2}$$

3) Constant coefficients : Three cases ← distinct, repeated, complex

$$ay'' + by' + cy = 0$$

4) Method of Undetermined coefficients

$$ay'' + by' + cy = g(t)$$

5) Method of variation of parameters

$$ay'' + by' + cy = g(t)$$

$$y_p = u_1(t)y_1(t) + u_2(t)y_2(t)$$

$$\begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix} \begin{pmatrix} u_1' \\ u_2' \end{pmatrix} = \begin{pmatrix} 0 \\ g(t) \end{pmatrix}$$

Chapter 7 Systems of ODEs

1) Abel's Formula for $X' = P(t)X$

$W' = \text{trace}(P(t))\, W$

2) Constant Coefficients $X' = AX$

Three cases ← distinct, repeated, complex

3) Method of Diagonalization

$X' = AX + g(t)$

4) Method of Undetermined Coefficients

5) Method of Variation of Parameters

6) Fundamental matrix

$\underline{\Psi}(t) = (X^{(1)} \cdots X^{(n)})$

$\underline{\Phi}(t) = (X^{(1)} \cdots X^{(n)})$ such that $\underline{\Phi}(0) = \underline{I}$

Chapter 6 Laplace Transform

$$f(t) \longrightarrow F(s) = \int_0^{+\infty} e^{-st} f(t)\, dt$$

(1) Important formulas for Laplace Transform

(Table will be given to you)

(2) Solve

$$ay'' + by' + cy = g(t)$$

with $g(t)$ being

- smooth functions of $e^{\alpha t} \cos \beta t\, Q(t)$
- piecewise smooth functions

$$\sum_{j=1}^{n} u_{c_j}(t) f_j(t)$$

- Dirac function: $f(t)\, \delta(t-c)$

(3) Method of Partial Fractions

Important: $u_c(t) f(t-c) \quad <\!\!> \quad e^{-cs} F(s)$

Chapter 10: Fourier series and method of separation of variables

(1) $f(x)$ periodic with $2L$ period

$$f(x+2L) = f(x)$$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{+\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right)$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx, \quad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

(2) $f(x)$ is odd or f satisfies $f(0) = f(L) = 0$

$$f(x) \sim \sum_{n=1}^{+\infty} b_n \sin\left(\frac{n\pi x}{L}\right) \quad b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

(3) $f(x)$ is even or f satisfies $f'(0) = f'(L) = 0$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{+\infty} a_n \cos\left(\frac{n\pi x}{L}\right), \quad a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

(4) Convergence of Fourier series

$$\frac{a_0}{2} + \sum_{n=1}^{+\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right) = \frac{1}{2}\left(f(x^-) + f(x^+)\right)$$

(5) Method of Separation of Variables

- heat equation

$$\begin{cases} u_t = \alpha^2 u_{xx} \\ u(x,0) = f(x) \\ u(0,t) = u(L,t) = 0 \end{cases}$$

$$u(x,t) = \sum_{n=1}^{+\infty} b_n e^{-\alpha^2 \left(\frac{n\pi}{L}\right)^2 t} \sin\left(\frac{n\pi}{L}x\right)$$

$$b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi}{L}x\right) dx$$

Neumann BC: $u_x(0,t) = u_x(L,t) = 0$

$$u(x,t) = \frac{a_0}{2} + \sum_{n=1}^{+\infty} a_n e^{-\alpha^2 \left(\frac{n\pi}{L}\right)^2 t} \cos\left(\frac{n\pi}{L}x\right)$$

$$a_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi}{L}x\right) dx$$

- Wave equation: $u_{tt} = a^2 u_{xx}$
- Laplace equation: $u_{xx} + u_{yy} = 0$