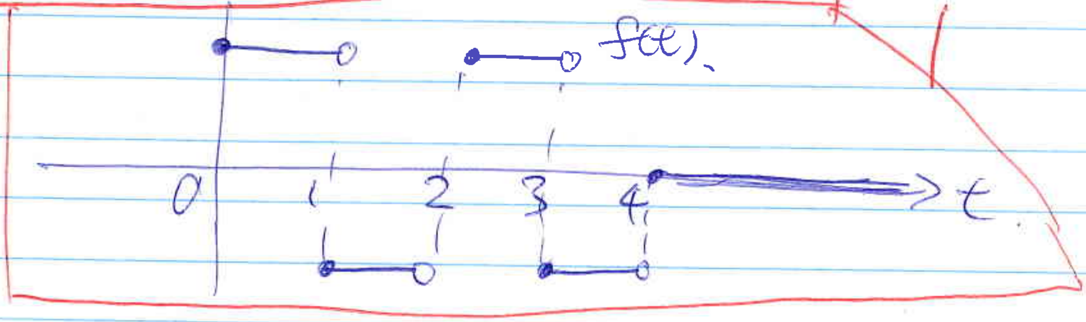
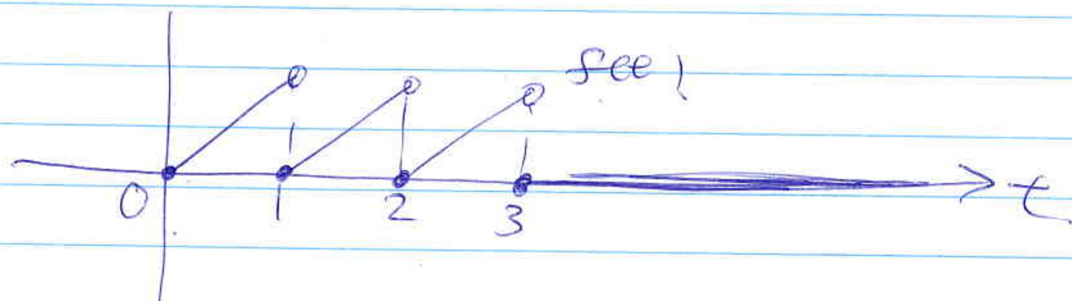
6.3. 8. $f(t) = 1 - 2u_1(t) + 2u_2(t) - 2u_3(t) + u_4(t)$



11. $f(t) = t - u_1(t) - u_2(t) - u_3(t)(t-2)$



15. $F(s) = \int_{\pi}^{2\pi} (t-\pi)e^{-st}\,dt = \dfrac{e^{-\pi s}}{s^2} - \dfrac{e^{-2\pi s}}{s^2}(1+\pi s)$.

19. $f(t) = t^3 e^{2t}$

23. $F(s) = \dfrac{(s-2)e^{-s}}{s^2-4s+3} = e^{-s}\dfrac{(s-2)}{(s-2)^2-1}$.

$f(t) = \mathcal{L}^{-1}\left(e^{-s}\dfrac{s-2}{(s-2)^2-1}\right) = u(t-1)\mathcal{L}^{-1}\left(\dfrac{s-2}{(s-2)^2-1}\right)(t-1)$.

$= u_1(t)e^{2(t-1)}\mathcal{L}^{-1}\left(\dfrac{s}{s^2-1}\right)(t-1)$

$= u_1(t)e^{2(t-1)}\cosh(t-1)$.

~~28.~~ $\cancel{F(s) = \dfrac{1}{9s^2-12s+3} = \dfrac{1}{3(3s^2-4s+1)} = \dfrac{1}{3(3s-}}$

$\cancel{= \dfrac{1}{9(s^2-\frac{4}{3}s+\frac{1}{3})} = \dfrac{1}{9\left((s-\frac{2}{3})^2-\frac{1}{9}\right)}}$.

$\cancel{f(t)=\mathcal{L}^{-1}\left(\frac{1}{9\left(\left(s-\frac{2}{3}\right)^2-\frac{1}{9}\right)}\right)=\frac{1}{4}}$

28. $f(t)=\mathcal{L}^{-1}\left(\frac{1}{(3s)^2-4(3s)+3}\right)(t)=\mathcal{L}^{-1}\left(\frac{1}{(3s-2)^2-1}\right)(t)$

$=\frac{1}{3}e^{\frac{2t}{3}}\mathcal{L}^{-1}\left(\frac{1}{s^2-1}\right)\left(\frac{t}{3}\right)$

$=\frac{1}{3}e^{\frac{2t}{3}}\cosh\left(\frac{t}{3}\right).$ □

29. $\cancel{F(s)}$ $f(t)=\mathcal{L}^{-1}\left(\frac{e^{-2(2s-1)}}{2s-1}\right)(t)$

$=\frac{1}{2}e^{\frac{t}{2}}\mathcal{L}^{-1}\left(\frac{e^{-2s}}{s}\right)\left(\frac{t}{2}\right)$

$=\frac{1}{2}e^{\frac{t}{2}}u_2\left(\frac{t}{2}\right)$ □

6.4. 2(a). $\begin{cases} y''+2y'+2y=u_\pi(t)-u_{2\pi}(t) \\ y(0)=0,\ y'(0)=1 \end{cases}$

$(s^2+2s+2)Y(s)-1=\frac{e^{-\pi s}-e^{-2\pi s}}{s}$

$Y(s)=\frac{1}{(s+1)^2+1}+\frac{e^{-\pi s}}{s[(s+1)^2+1]}-\frac{e^{-2\pi s}}{s[(s+1)^2+1]}$

$$\boxed{\begin{aligned} y(t)=\mathcal{L}^{-1}(Y(s))(t) &= e^{-t}\sin t+\frac{1}{2}u_\pi(t)\left[1+e^{-(t-\pi)}\cos t+e^{-(t-\pi)}\sin t\right] \\ &\quad -\frac{1}{2}u_{2\pi}(t)\left[1-e^{-(t-2\pi)}\cos t-e^{-(t-2\pi)}\sin t\right] \end{aligned}}$$ □

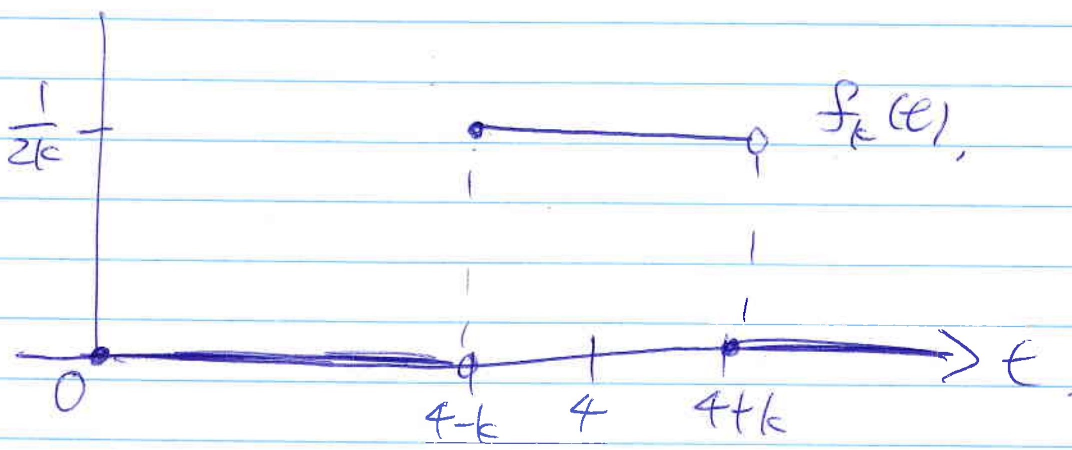
4(a). Similar.

$y=\frac{1}{6}(2\sin t-\sin 2t)-\frac{1}{6}u_\pi(t)(2\sin t+\sin 2t)$ □

9.(a) $y = \frac{1}{2}(\sin t + t) - \frac{1}{2}u_6(t)\left(t - 6 - \sin(t-6)\right)$.

10(a). $y = h(t) + u_\pi(t)\, h(t-\pi)$,
where $h(t) = \frac{4}{17}\left[-4\cos t + \sin t + 4e^{-\frac{t}{2}}\cos t + e^{-\frac{t}{2}}\sin t\right]$

18.(a)



(b). $f_k(t) = \dfrac{u_{4-k}(t) - u_{4+k}(t)}{2k}$

$$y = \frac{1}{2k}\left[u_{4-k}(t)\,h(t-4+k) - u_{4+k}(t)\,h(t-4-k)\right]$$

where $h(t) = \frac{1}{4} - \frac{1}{4}e^{-\frac{t}{6}}\cos\frac{\sqrt{143}\,t}{6} - \frac{\sqrt{143}}{572}e^{-\frac{t}{6}}\sin\frac{\sqrt{143}\,t}{6}$

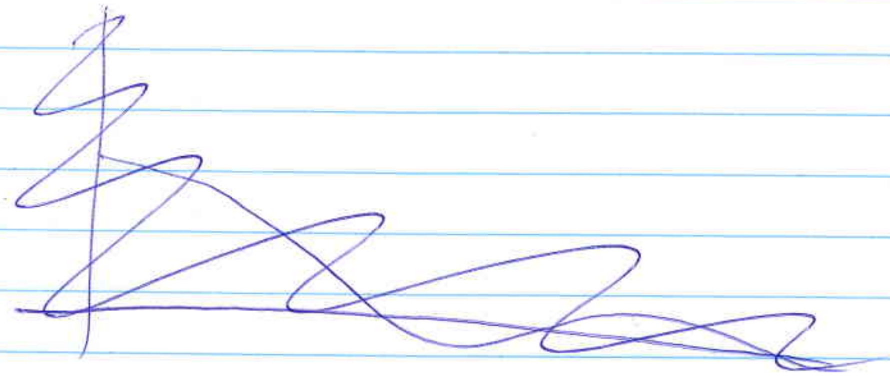
<u>6.5</u> 1.(a). $\begin{cases} y'' + 2y' + 2y = \delta(t-\pi) \\ y(0) = 1,\ y'(0) = 0 \end{cases}$

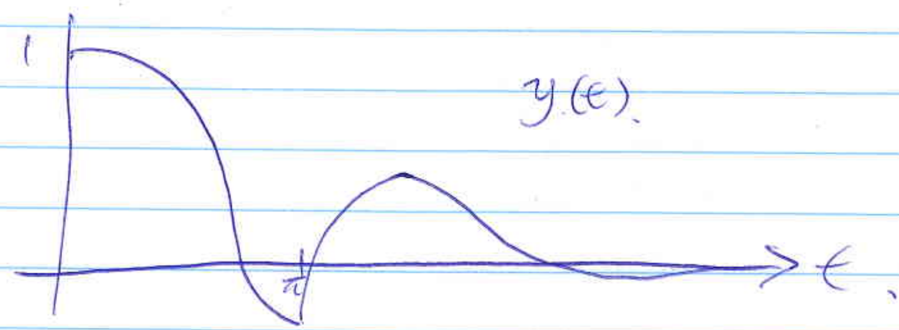
~~解~~ $(s^2+2s+2)Y(s) - s - 2 = e^{-\pi s}$

$$Y(s) = \frac{s+1}{(s+1)^2+1} + \frac{1}{(s+1)^2+1} + \frac{e^{-\pi s}}{(s+1)^2+1}$$

$$y = e^{-t}\cos t + e^{-t}\sin t + u_\pi(t)\,e^{-(t-\pi)}\sin(t-\pi)$$

1.(b).



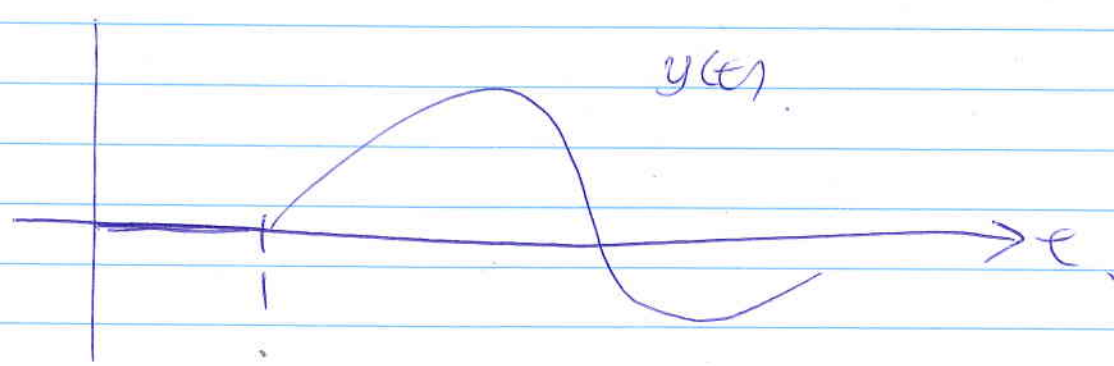


5.(a) Similar.

$$\boxed{y=\frac{1}{4}\left(\sin t-\cos t+e^{-t}\cos\sqrt{2}\,t\right)+\frac{1}{\sqrt{2}}u_{3\pi}(t)e^{-(t-3\pi)}\sin\sqrt{2}(t-3\pi)}$$

10(a). $y=\frac{1}{\sqrt{31}}u_{\frac{\pi}{6}}(t)e^{-\frac{t-\frac{\pi}{6}}{4}}\sin\frac{\sqrt{31}}{4}\left(t-\frac{\pi}{6}\right)$.

14.(a) $y=\frac{4}{\sqrt{15}}u_1(t)e^{-\frac{t-1}{4}}\sin\frac{\sqrt{15}}{4}(t-1)$.



(b) $t_1\approx 2.3613$, $y_1\approx 0.71153$.

16 (a). $$\boxed{\phi(t,k) = \frac{1}{2k}\left[u_{4-k}(t)h(t-4+k) - u_{4+k}(t)h(t-4-k)\right] \quad \text{where } h(t) = 1-\cos t.}$$

(b). $\phi_0(t) = u_4(t)\sin(t-4)$. □

6.6 4. $\mathcal{L}\left(\int_0^t (t-\tau)^2\cos 2\tau\, d\tau\right) = \mathcal{L}\left((u(t)t^2) * \cancel{(\cos 2t)}\right)$ $(u(t)\cos(2t))$

$= \mathcal{L}(u(t)t^2)\,\cancel{\mathcal{L}(\cos 2t)}\,\mathcal{L}(u(t)\cos(2t))$.

$= \frac{2}{s^3}\frac{s}{s^2+4} = \frac{2}{s^2(s^2+4)}$. □

6. Similar.

$F(s) = \frac{1}{s^2(s-1)}$. □

10. $\mathcal{L}^{-1}\left(\frac{1}{(s+1)^2(s^2+4)}\right) = \mathcal{L}^{-1}\left(\frac{1}{(s+1)^2}\right) * \mathcal{L}^{-1}\left(\frac{1}{s^2+4}\right)$.

$= \boxed{\frac{1}{2}\int_0^t (t-\tau)e^{-(t-\tau)}\sin 2\tau\, d\tau.}$ □

13. $\begin{cases} y'' + \omega^2 y = g(t) \\ y(0)=0,\ y'(0)=1 \end{cases}$

$(s^2+\omega^2)Y(s) - 1 = \mathcal{L}(g(t))(s)$.

$Y(s) = \frac{1}{\omega}\left(\frac{\omega}{s^2+\omega^2} + \frac{\omega}{s^2+\omega^2}\mathcal{L}(g(t))(s)\right)$.

$y(t) = \frac{1}{\omega}\sin\omega t + \frac{1}{\omega}\int_0^t \sin\omega(t-\tau)g(\tau)d\tau$. □

1f. Similar,

$$\boxed{y = 2e^{-t} - e^{-2t} + \int_0^t \left(e^{-(t-\tau)} - e^{-2(t-\tau)}\right)\cos\alpha\tau \, d\tau}$$

$\square$