Math 257/316 Assignment 4

Due Friday Feb. 6 in class

1. Consider the heat conduction problem:

$$\frac{\partial u}{\partial t} = 5\frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 3,\ t > 0,$$

with homogeneous boundary conditions

$$u(0,t) = u(3,t) = 0.$$

Find the solution for each of the initial conditions (using formulas from class/notes/text if you like):

a) $u(x,0) = 4\sin \pi x$

b) $u(x,0) = \sin(\pi x/3) - 2\sin(2\pi x/3) + 11\sin(2\pi x)$

2. Use the method of separation of variables to find the most general solution of the following heat conduction problem with "mixed" boundary conditions:

$$\begin{aligned} &u_t = \alpha^2 u_{xx}, \quad 0 < x < L,\ t > 0, \\ &u(0,t) = 0, \ \ u_x(L,t) = 0. \end{aligned}$$

3. Use the method of separation of variables to solve the problem

$$\begin{aligned} &u_t = u_{xx} + au, \quad 0 < x < 1,\ t > 0, \\ &u(0,t) = 0, \ \ u(1,t) = 0, \\ &u(x,0) = \sin(\pi x) \end{aligned}$$

How does the long term ($t \to \infty$) behaviour of the solution depend on the constant a?