

Math 257/316 Assignment 5

Due Monday Feb. 23 in class

1. For the “sawtooth” function

$$f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2 - x & 1 \leq x \leq 2 \end{cases}$$

defined on $[0, 2]$, compute its

- (a) compute its Fourier sine series
- (b) compute its Fourier cosine series
- (c) by evaluating $f(1)$, use each of your results from (a) and (b) in turn, to find the value of the sum of the squares of the reciprocals of the odd integers:

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \cdots .$$

2. Solve the heat conduction problem:

$$\begin{aligned} u_t &= 2u_{xx}, & 0 < x < 2, & t > 0, \\ u(0, t) &= 0 = u(2, t) \\ u(x, 0) &= \begin{cases} x & 0 \leq x \leq 1 \\ 1 - x & 1 \leq x \leq 2 \end{cases}, & 0 \leq x \leq 2. \end{aligned}$$

3. Find the solution of the following problem describing the temperature in a wire with insulated ends:

$$\begin{aligned} u_t &= 3u_{xx}, & 0 < x < \pi, & t > 0, \\ u_x(0, t) &= 0 = u_x(2, t) \\ u(x, 0) &= x, & 0 \leq x \leq 2. \end{aligned}$$

4. Solve the following problem describing heat conduction in a closed thin circular wire:

$$\begin{cases} u_t = u_{xx}, & -1 < x < 1, & t > 0, \\ u(-1, t) = u(1, t), & u_x(-1, t) = u_x(1, t), \\ u(x, 0) = |x|, & -1 \leq x \leq 1 \end{cases}$$