Assignment 4 - MATH 301-201, Due Date: March 9, 2016

1. Find the image of $|z-2|=1$ and its interior under the following transforms

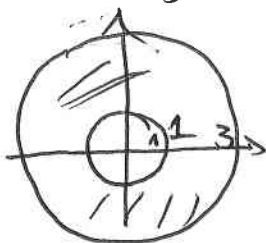   (a) $w = \dfrac{z-2}{z-1}$, (b) $= \dfrac{z-4}{z-3}$

2. Find Mobius transforms so that $0, 1, \infty$ are mapped to

   (a) $0, i, \infty$,   (b) $-i, \infty, 1$ (c) $1, i, \infty$

3. What is the image of the third quadrant under the map $w = \dfrac{z+i}{z-i}$?

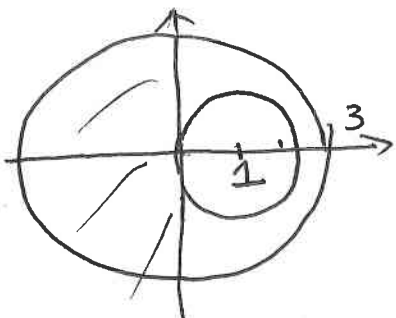4. Solve the following Laplace equation

   (a)

   $$\phi_{xx} + \phi_{yy} = 0, \qquad |z| < 3, \; |z| > 1$$
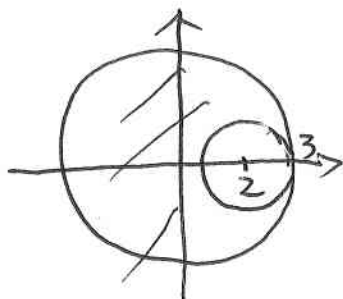 $$\phi = 1, \qquad |z| = 3$$
   $$\phi = 2, \qquad |z| = 1$$

   (b)

   $$\phi_{xx} + \phi_{yy} = 0, \quad |z| < 3, \; |z-1| < 1$$
   $$\phi = 1, \quad |z| = 3$$
   $$\phi = 2, \quad |z-1| < 1$$

   (c)

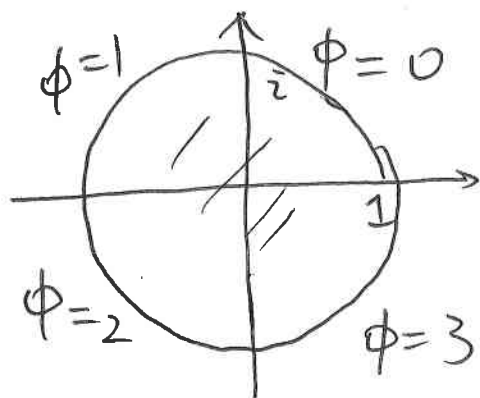 $$\phi_{xx} + \phi_{yy} = 0, \quad |z| < 3, \; |z-2| > 1$$
   $$\phi = 1, \quad |z| = 3$$
   $$\phi = 2, \quad |z-2| > 1$$

   (d)

   $\phi = 1$ $\phi = 0$

   $\phi = 2$ $\phi = 3$

   $$\phi_{xx} + \phi_{yy} = 0, \qquad |z| < 1$$
   $$\phi = \begin{cases} 0, & |z| = 1, \; 0 < \varphi < \frac{\pi}{2} \\ 1, & |z| = 1, \; \frac{\pi}{2} < \varphi < \pi \\ 2, & |z| = 1, \; \pi < \varphi < \frac{3\pi}{2} \\ 3, & |z| = 1, \; \frac{3\pi}{2} < \varphi < 2\pi \end{cases}$$