Remark on Problem 3: You will need to evaluate an integral of the following form

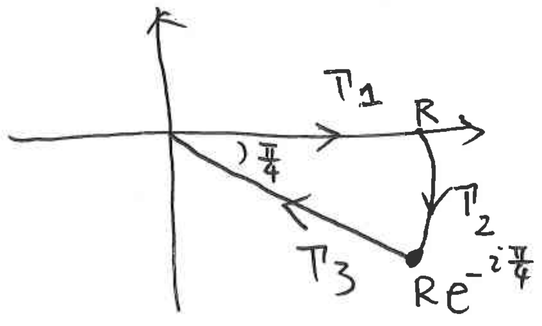
$$\int_{-\infty}^{+\infty} e^{-itx^2}\,dx = 2\int_0^{+\infty} e^{-itx^2}\,dx$$

$$= \frac{2}{t^{\frac{1}{2}}} \int_0^{+\infty} e^{-ix^2}\,dx$$

the value of $\int_0^{+\infty} e^{-ix^2}\,dx$ can be computed using complex contour.

Claim: $\int_0^{+\infty} e^{-ix^2}\,dx = \frac{\sqrt{\pi}}{2}\, e^{-i\pi/4}$

Proof:



$f(z) = e^{-iz^2}$

$$\int_{T_1} f(z)\,dz = \int_0^R e^{-ix^2}\,dx$$

$$\int_{T_3} f(z)\,dz = \int_R^0 e^{-i(e^{-i\frac{\pi}{4}}p)^2} e^{-i\frac{\pi}{4}}\,dp$$

$$= -e^{-i\frac{\pi}{4}} \int_0^R e^{-p^2}\,dp$$

$\int_{T_2} f(z)\,dz \to 0$ as $R \to +\infty$

So as $R \to \infty$ $\int_0^{+\infty} e^{-ix^2}\,dx = e^{-i\frac{\pi}{4}} \int_0^{\infty} e^{-p^2}\,dp = \frac{\sqrt{\pi}}{2}\, e^{-i\frac{\pi}{4}}$