Assignment 6 — MATH301-201 (Due Date: April, 2016)

1. Consider the complex velocity potential $\Omega(z)$ given by

$$\Omega(z) = V_0\left(z + \frac{a^2}{z}\right) + i\frac{\gamma}{2\pi}\operatorname{Log} z \quad \text{with } \gamma > 0$$

(a) Find the velocity $v(z)$

(b) Find the stagnation points. Determine the critical γ_c

(c) Draw streamlines for $\gamma < \gamma_c$ and $\gamma > \gamma_c$

2. Find out the Fourier Transforms of

(a) $f(t) = \dfrac{2}{t^2+4}$ (b) $f(t) = e^{-2|t|}$

(c) $f(t) = \dfrac{1}{t^4+1}$ (d) $f(t) = \dfrac{1}{(t^2+1)^2}$

(e) $f(t) = e^{-t^2}$ (f) $f(t) = t\,e^{-t^2}$

3. Use Fourier Transform to solve the Schrödinger Equation

$$iu_t + u_{xx} = 0, \quad -\infty < x < +\infty, \quad t > 0$$

$$u(x,0) = f(x), \quad f(x) \to 0 \text{ as } |x| \to \infty$$

4. Use Fourier Transform to solve the diffusion equation in R^2

$$u_t = D(u_{xx} + u_{yy}), \quad -\infty < x < +\infty, \quad -\infty < y < +\infty$$

$$u(x,y,0) = f(x,y) \to 0 \text{ as } |x|+|y| \to +\infty$$

5. For the following problem, find the dispersion relation and find general solution

$$\begin{cases} u_t = D_0 u_{xx} - D_1 u_{xxxx} \\ u(x,0) = f(x) \end{cases}, \quad D_0, D_1 > 0$$