MATH301-201 Homework Assignment 7 (Due Date: by 6pm, April 13, 2016)

Please either hand in to my office or send it by email by 6pm of April 13, 2016. The solutions will be put on my web-site on April 14.

1. Find the inverse Laplace transform of

$$F(s) = e^{-a\sqrt{s}}, a > 0$$

Hint: use the complex contour as in the Lecture notes.

2. Solve the following second order ODE using Laplace transform

$$u'' - 2u' + 5u = 1 + 5\sin(t), \; u(0) = a, u'(0) = 1$$

Find the unique initial value a such that u is bounded.

3. Use the Laplace transform to find the general formula for

$$y''' + y = f(t)$$

Hint: The inverse Laplace transform of $\hat{g}(s)\hat{f}(s)$ is $\int_0^t g(t-\tau)f(\tau)d\tau$.

4. (a) Suppose that $f(t+T) = -f(t)$. Show that the Laplace transform

$$\hat{f}(s) = \frac{\int_0^T e^{-st} f(t)dt}{1 + e^{-sT}}$$

(b) Find the Laplace transform of the following function

$$f(t) = \begin{cases} 1, \; t \in (0,T) \cup (2T, 3T) \cup (4T, 5T) \cup ..., \\ -1, \; t \in (T, 2T) \cup (3T, 4T) \cup (5T, 6T) \cup ... \end{cases}$$

5. Consider $y' + y = f(t)$ with $f(t+1) = f(t)$ with initial value $y(0) = y_0$. Here $f(t) = 0$ for $0 \leq t < 1/2$ and $f(t) = 1$ for $1/2 \leq t < 1$. Find the special value y_0 such that the solution y is also periodic $y(t+1) = y(t)$.

6. Use the Laplace transform to find the solution to the diffusion equation

$$u_t = u_{xx}, 0 < x < \infty, t > 0$$

$$u(x,0) = e^{-x}, 0 < x < \infty$$

$$u(0,t) = 1, t > 0$$

Hint: The inverse Laplace transform of $\frac{e^{-a\sqrt{s}}}{s}$ is $1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{a}{2\sqrt{t}}} e^{-p^2} dp = \int_{\frac{a}{2\sqrt{t}}} e^{-p^2} dp$, and the inverse transform of $e^{-a\sqrt{s}}$ is given in Problem 1.

7. Find the number of zeroes of $f(z) = z^3 + 2z^2 + z + 1$ in the right-half plane.