Midterm Examination for MATH301-201

Total: 100 points

Answer All Questions. Show All Steps.

Date: Feb. 26th. 2016

1. (20pts) Evaluate the integral $\int_0^\infty \frac{\log x}{x^3+1}dx$

2. (20pts) Consider the following multi-valued function

$$w = f(z) = (\frac{z+1}{z-1})^{\frac{1}{2}}$$

(a) Define a branch that is continuous at $z = \pm 2$ with $f(2) = \sqrt{3}$ and evaluate $f(-2)$.

(b) Define a different branch with $f(2) = \sqrt{3}$ that is continuous at $z = \pm 2$ and at the origin.

3. (30pts) Identify the contours and the functions $f(z)$ to compute the following integrals. **Don't attempt to compute the precise values of the integral.** The contours are listed on the other page.

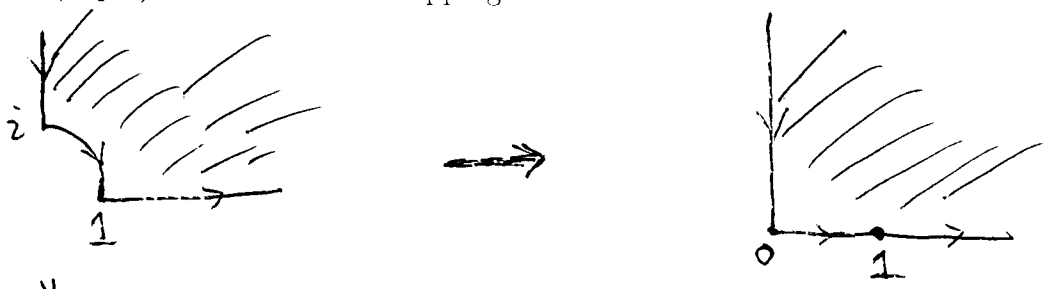
$$(a)\ \int_{-\infty}^{+\infty} \frac{\cos(x)}{x^2+2x+2}dx, \qquad (b)\ \int_{-\infty}^{+\infty} \frac{e^{-ix}}{x-i}dx$$

$$(c)\ \int_0^{+\infty} \frac{\log x}{x^2+4}dx, \qquad (d)\ \int_0^{+\infty} \frac{1}{x^2+2x+2}dx$$

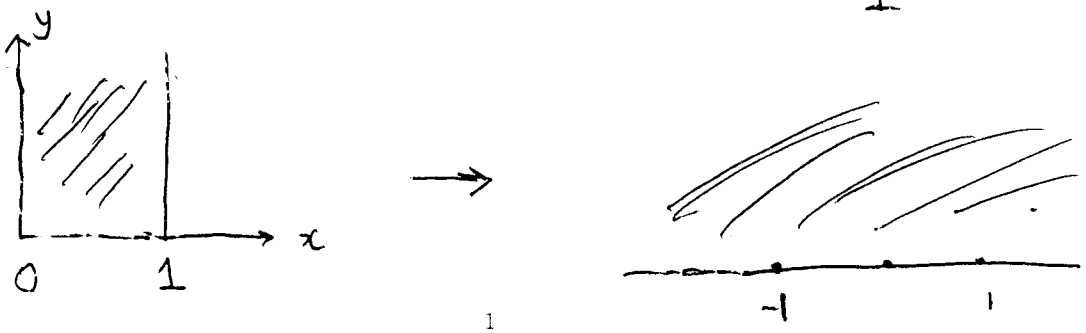
$$(e)\ \int_0^1 \frac{1}{x^{\frac{1}{3}}(1-x)^{\frac{2}{3}}}dx$$

4. (30pts) Find conformal mappings:

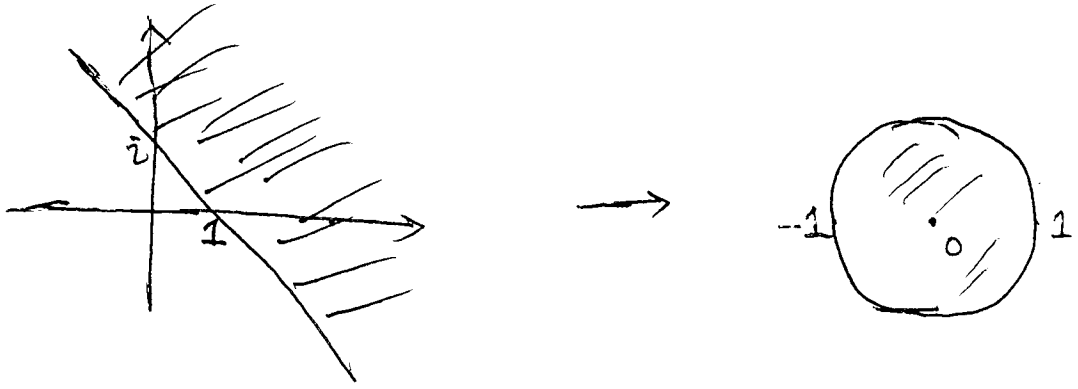
(a)



(b)

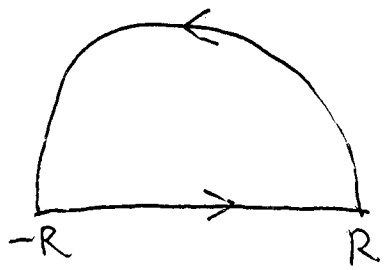


(c)

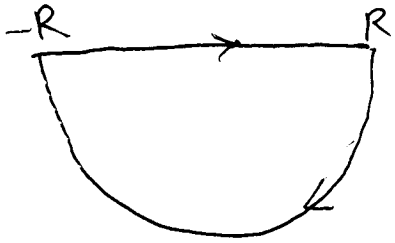


Contours

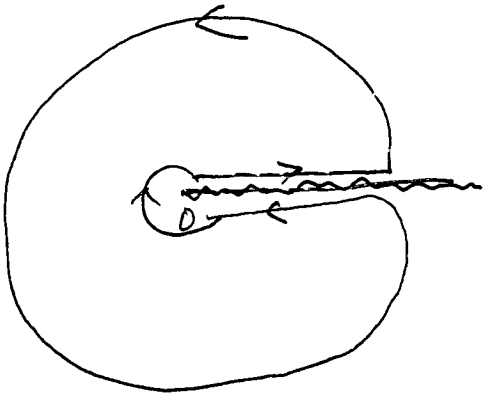
1



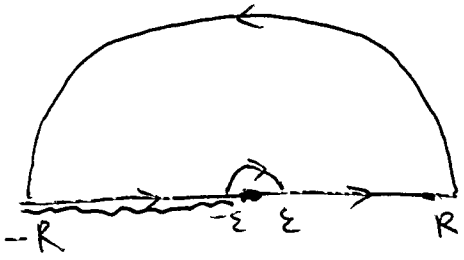
2.



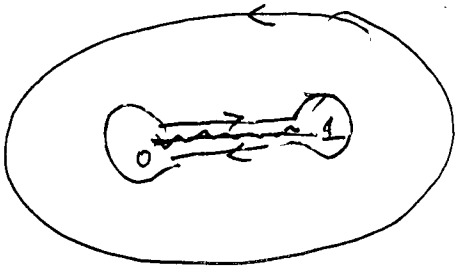
3.



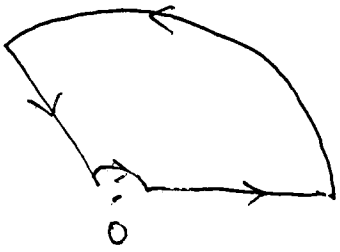
4.



5.



6.



7.