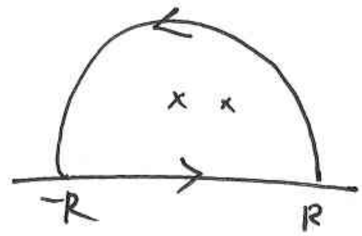
# A Summary of Real Integral Computations using Complex Contours

**Type I.** $\displaystyle\int_0^{2\pi} R(\cos\theta, \sin\theta)\,d\theta \;\underline{\underline{z=e^{i\theta}}}\; \int_{|z|=1} f(z)\,dz$

$|z|=1$

**Type II** II.1 $\displaystyle\int_{-\infty}^{+\infty} \frac{P(x)}{Q(x)}\,dx$ or $\displaystyle\int_0^{+\infty} \frac{P(x)}{Q(x)}\,dx$, $P, Q$ are even

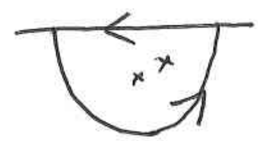
$\deg Q \geq \deg P + 2$




II.2. $\displaystyle\int_0^{+\infty} R(x^m)\,dx$

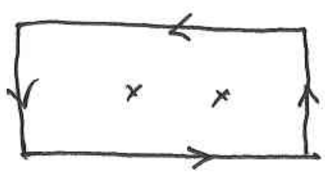

**Type III** $\displaystyle\int_{-\infty}^{+\infty} \frac{P(x)}{Q(x)} e^{i\beta x}\,dx$

$\beta > 0$

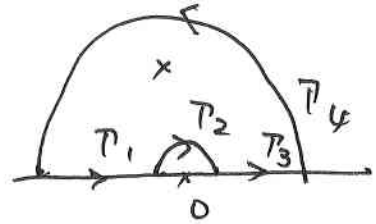


$\beta < 0$

**Type IV** $\displaystyle\int_{-\infty}^{+\infty} R(e^x)\,dx$

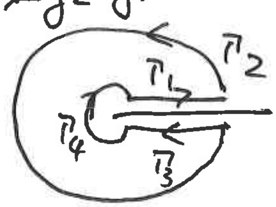


**Type V**




V.1 $\displaystyle\int_0^{+\infty} x^\alpha g(x)\,dx$, or $\displaystyle\int_0^{+\infty} \log x\; g(x)\,dx$, $g$ even.

~~V.2~~ $f(z) = z^\alpha g(z)$ or $f(z) = \log z\; g(z)$




V.2 $\displaystyle\int_0^{+\infty} x^\alpha g(x)\,dx$, $g$ general

$f(z) = z^\alpha g(z)$


V.3 $\displaystyle\int_0^{+\infty} \log x\; g(x)\,dx$, $g$ general, $f(z) = \log^2 z\; g(z)$

V.4 $\displaystyle\int_0^{+\infty} g(x)\,dx$, $g$ general, $f(z) = \log z\; g(z)$

V.5. $\displaystyle\int_0^{+\infty} \log^k x\; g(x)\,dx$, $g$ general, $f(z) = \log^{k+1} z\; g(z)$ ↗