MATH305-201-2016/2017 Homework Assignment 4 (Due Date: Feb. 6, 2017, by 5:30pm, in class or at my office)

1. Find an analytic mapping from $\{-1 < x < 3, y > -1\}$ onto the upper half plane $\{v > 0\}$. Hint: consider the map $\sin(z)$.

2. Evaluate the following

(a) $log(i)$; (b) $Log(\sqrt{3} + i)$; (c) $log(e^i)$; (d) $e^{log(i)}$

3. Find all values of

(a) $e^z = -1 - 2i$; (b) $\sin(z) = 1$; (c) $(1 + i)^{\frac{1}{3}}$; (d) i^i

4. Solve the following equations

(a) $Log(z^2 - 1) = \frac{i\pi}{2}$; (b) $e^{2z} + e^z + 1 = 0$; (c) $z^{\frac{1}{2}} + 1 - i = 0$ (here $z^{\frac{1}{2}}$ denotes the principal branch)

5. Determine the domain of analyticity (branch cut) of

(a) $Log(1 + z^2)$; (b) $Log(\frac{1-z}{1+z})$; (c) $Log(e^z)$

6. Which of the followings are true statements? For the ones that are true provide a proof. For the ones that are false find a counterexample

(a) $e^{log(z)} = z$; (b) $e^{Log(z)} = z$; (c) $Log(e^z) = z$; (d) $log(e^z) = z$; (e) $Log(z_1 z_2) = Log(z_1) + Log(z_2)$; (f) $log(z_1 z_2) = log z_1 + log z_2$; (g) $log(z) = -log(\frac{1}{z})$; (h) $log(z^{\frac{1}{2}}) = \frac{1}{2} log(z)$

7. Find a branch cut of $log(2z - 1)$ that is analytic at all points in the plane except those on the following rays.

(a) $\{x \leq \frac{1}{2}, y = 0\}$; (b) $\{x \geq \frac{1}{2}, y = 0\}$; (c) $\{x = \frac{1}{2}, y \geq 0\}$

8. Find a one-to-one analytic mapping of the upper half plane $\{Im(z) > 0\}$ onto the stripe $\{-\infty < u < +\infty, 0 < v < 1\}$.

9. Determine a branch of $log(z^2 + 2z + 3)$ that is analytic at $z = -1$, and find its derivative there.

10. Determine a branch of $log(1 + z^2)$ that is analytic at $z = 0$ and takes the value $2\pi i$ there.

11. Find a branch cut for $\sqrt{z(z-1)}$ that is analytic in $C \backslash [0, 1]$.