MATH305-201-2016/2017 Homework Assignment 5 (Due Date: Feb. 15, 2017, by 5:30pm, in class or at my office)

1. Find a branch cut for $f(z) = (z^3 - z)^{\frac{1}{2}}$ such that $f(-\frac{1}{2}) = \sqrt{\frac{3}{8}}$ (so f is analytic at $-\frac{1}{2}$).

2. Find the region where $f(z) = Log(1 - z^3)$ is analytic.

3. Find a branch of each of the following multi-valued functions that is analytic in the given domain
 (a) $(4 + z^2)^{\frac{1}{2}}$ in $C \backslash \{x = 0, -2 \leq y \leq 2\}$; (b) $(z^4 - 1)^{\frac{1}{2}}$ in $\{|z| > 1\}$.

4. Find all solutions to
 (a) $\cosh(z) = i$; (b) $\sin(z) = i + 1$; (c) $\cos(z) = 2i$; (d) $(e^z - 1)^3 = 1$

5. Find a solution to the boundary value problem and evaluate it at $(2, 3)$:

$$u_{xx} + u_{yy} = 0, y > 0, -\infty < x < +\infty$$

$$u(x, 0) = \begin{cases} 0, x < -1; \\ \pi, \ -1 < x < 2; \\ -\pi, \ x > 2 \end{cases}$$

6. Find a solution to the boundary value problem and evaluate it at $(0, 0)$:

$$u_{xx} + u_{yy} = 0, \quad 1 < (x - 1)^2 + (y - 1)^2 < 4$$

$$u = 1 \text{ on } (x - 1)^2 + (y - 1)^2 = 1; u = 5 \text{ on } (x - 1)^2 + (y - 1)^2 = 4$$

7. Find a solution to the boundary value problem and evaluate it at $(0, 2)$

$$u_{xx} + u_{yy} = 0, y > 1, y > -x$$

$$u(x, 1) = 1 \text{ for } x > -1; u(x, -x) = 2 \text{ for } x < -1$$

8. Find an inverse function for $sinh(z) = \frac{e^z - e^{-z}}{2}$ such that its value at 0 equals 0.

9. Prove that $|\sin(z)| \leq 3$ when $|z| \leq 1$.

10. Compute the integral $\int_C f dz$ using the contour (always counter-clockwise) given
 (a) $f = x - 2xyi$; $C = \{y = x^2, 0 \leq x \leq 1\} \cup \{y = 1, -1 \leq x \leq 1\}$; (b) $f = \bar{z}^2$; C: square with vertices $z = 0, z = 1, z = 1 + i$ and $z = i$; (c) $f = Log(z)$; $C = \{|z| = 1, Re(z) \geq 0\}$