MATH305-201-2016/2017 Homework Assignment 6 (Due Date: Feb. 27, 2017, by 5:30pm, in class or at my office)

1. Evaluate $\int_\Gamma (2z+1)dz$, where Γ is the following contour from $z=-i$ to $z=1$:
(a) the simple line segment; (b) two simple line segments, the first from $z=-i$ to $z=0$ and the second from $z=0$ to $z=1$; (c) the circular arc $z=e^{it}, -\frac{\pi}{2}\le t\le 0$

2. Evaluate $\int_\Gamma \bar{z}dz$, where
(a) Γ is the circle $|z|=2$ traversed once counterclockwise; (a) Γ is the circle $|z|=2$ traversed twice counterclockwise; (a) Γ is the circle $|z|=2$ traversed three times clockwise

3. Evaluate $\int_C \frac{1}{z}dz$, where C is the contour defined in polar coordinate $z=re^{i\theta}$ by $r=3-\sin^2(\frac{\theta}{8}), 0\le\theta\le 8\pi$.
Hint: check page I10 of Lecture Note 5.

4. Use Fundamental Theorem of Calculus to evaluate
(a) $\int_C e^z dz$, C : arc $e^{it}, -\frac{\pi}{2}\le t\le \pi$; (b) $\int_C \frac{1}{z}dz$, C : part of the ellipse $\frac{x^2}{4}+y^2=1, x\ge 0$; (c) $\int_C \frac{1}{z^2}dz$, C : part of the ellipse $\frac{x^2}{4}+y^2=1, y\ge 0$.

5. Use the inequality $|\int_\Gamma f(z)dz|\le \max_{z\in\Gamma}|f(z)|\times$ length of(Γ) to prove
(a) $|\int_C \frac{dz}{z^2-i}|\le\frac{3\pi}{4}$, C: circle $|z|=3$ traversed once; (b) $|\int_C Log(z)dz|\le\frac{\pi^2}{\mathbf{2}}$, C : arc $e^{it}, -\frac{\pi}{2}\le t\le\frac{\pi}{2}$; (c) $|\int_C \frac{e^{3z}}{e^z+1}dz|\le\frac{2\pi e^{3R}}{e^R-1}$, C is the vertical line segment from $z=R(>0)$ to $z=R+2\pi i$.
Hint: Check page I12 and page I13 of Lecture Note 5.

6. Show that
(a) $\int_{C_\epsilon}\frac{Log(z)}{1+z^2}dz\to 0$ as $\epsilon\to 0$, where C_ϵ is the contour $\epsilon e^{it}, -\pi+\epsilon\le t\le\pi-\epsilon$; (b) $\int_{C_R}\frac{Log(z)}{1+z^2}dz\to 0$ as $R\to+\infty$, where C_R is the contour $\epsilon e^{it}, -\pi+\frac{1}{R}\le t\le\pi-\frac{1}{R}$;

7. Use Fundamental Theorem of Calculus to compute
(a) $\int_\Gamma z^{\frac{1}{2}}dz$ for the principal branch of $z^{\frac{1}{2}}$, where Γ is $r=2\cos\frac{\theta}{2}, -\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$; (b) $\int_\Gamma (Log(z))^2dz$, where Γ is $r=2\cos\frac{\theta}{2}, -\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$
Hint: Check $(zz^{\frac{1}{2}})'=\frac{3}{2}z^{\frac{1}{2}}, (z(Log(z))^2)'=(Log(z))^2+2Log(z)$.

8. Let C be the contour of ellipse $\frac{x^2}{4}+y^2=1$ traversed once. Compute
(a) $\int_C\frac{1}{(z-1)^2}dz$; (b) $\int_C\frac{e^z}{z(z-1)}dz$; (c) $\int_C\frac{1}{z(z^2-1)}dz$; (d) $\int_C\frac{1}{2z^2+1}dz$