

MATH305 HW#7, 2016-2017 Due March 6, 2017, by 5:30pm

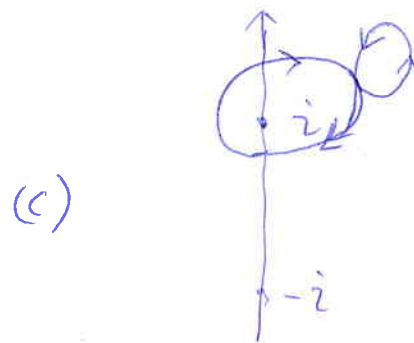
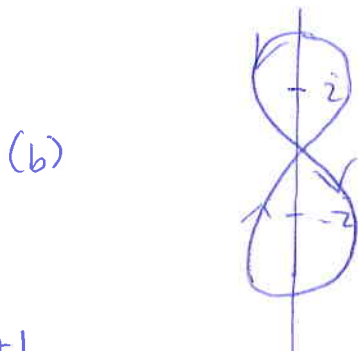
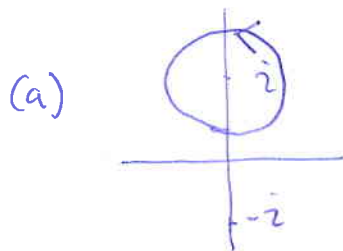
1. Determine the domain of analyticity for each of the given function f and explain why

$$\int_{|z|=2} f(z) dz = 0$$

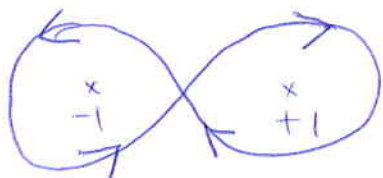
(a) $f(z) = \frac{\cos z}{z^2 - 6z + 10}$

(b) $f(z) = \log(2z+5)$, (c) $f(z) = \sec\left(\frac{z}{2}\right)$

2. Evaluate $\int_C \frac{1}{z^2+1} dz$ along the three contours



3. Evaluate $\int_{\Gamma} \frac{2z^2 - z + 1}{(z-1)^2(z+1)} dz$ along the figure 8 contour

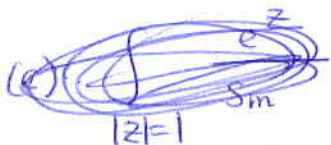


Hint: Use partial fraction expansion $\frac{A}{(z-1)^2} + \frac{B}{(z-1)} + \frac{C}{z+1}$

4. Evaluate

(a) $\int_{|z|=2} \frac{dz}{z^2 + 2z + 2}$

(b) $\int_{|z-1|=1} \frac{\sin z}{2z^2 - 5z - 3} dz$



5. Show that

$$\int_{|z|=5} \frac{1}{z^3 + 2z^2 + z + 1} dz = 0$$

Hint: show that $\int_{|z|=R} \frac{1}{z^3 + 2z^2 + z + 1} dz \rightarrow 0$ as $R \rightarrow \infty$

6. Evaluate

(a) $\int_0^{2\pi} \frac{d\varphi}{3 + \cos\varphi}$

(b) $\int_0^{2\pi} \sin^{10}\varphi d\varphi$

7. Evaluate

(a) $\int_{|z|=2} \frac{5z^2 + 2z + 1}{(z-1)^3} dz$

(b) $\int_{|z|=2} \frac{\sin z}{z^2(z-3)} dz$

8. Evaluate $\int_{|z|=3} \frac{e^{iz}}{(z^2+1)^2} dz$

9. Let f be analytic inside and on the unit circle $|z|=1$. Prove that if $|f(z)| \leq M$ for $|z|=1$, then

$$|f(0)| \leq M \text{ and } |f'(0)| \leq M.$$

What estimate can you give for $|f^{(n)}(0)|$?

10. Suppose that $f(z)$ is entire and $|f(z)| \leq |z|^4$. Show that $f(z)$ is a polynomial of degree at most 4.