MATH 305-201, 2016-2017, HW#9, Due Date: March 20, 2017, by 5:30 pm

1. Use Rouche's Theorem to find the number of zeroes for

(a) $p(z) = 6z^4 + z^3 - 2z^2 + z + 1$, in $|z| < 1$

(b) $p(z) = z^6 - 5z^2 + 10$ in $1 < |z| < 2$

2. Determine all the isolated singularities of each of the following functions and compute the residue at each singularity

(a) $\frac{z+1}{z^3 - 3z + 2}$, (b) $\frac{\cos z}{z^2}$, (c) $\frac{e^z}{z(z+1)^3}$, (d) $\sin\left(\frac{1}{3z}\right)$, (e) $\frac{z^2}{1 - \sqrt{z}}$

($\sqrt{z}$ denotes the principal branch)

3. Evaluate each of the following integrals by Cauchy Residue Theorem

(a) $\int_{|z|=3} \frac{e^z}{z(z-2)^3} dz$, (b) $\int_{|z|=2\pi} \tan z \, dz$, (c) $\int_{|z|=1} \frac{1}{z^2 \sin z} dz$

(d) $\int_{|z|} e^{\frac{1}{z}} \sin\left(\frac{1}{z}\right) dz$ (e) $\int_{|z|=1} \frac{e^z}{\sin^3 z} dz$

4. Calculate

(a) $\int_0^{\pi} \frac{8 \, d\theta}{5 + 2\cos\theta}$ (b) $\int_0^{\pi} \frac{1}{1 + \sin^2\theta} d\theta$ (c) $\int_0^{2\pi} \frac{\sin^2\theta}{3 + \cos\theta} d\theta$

5. Calculate

(a) $\int_0^{+\infty} \frac{x^2}{(x^2+9)^2} dx$ (b) $\int_0^{+\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ (c) $\int_0^{+\infty} \frac{1}{x^4 + x^2 + 1} dx$

(d) $\int_0^{+\infty} \frac{dx}{x^3 + 1}$