MATH400-201 Homework Assignment 4 (Due Date: March 8, 2016)

1. (20pts) Solve $u_{tt} = c^2 u_{xx}$ for $0 < x < +\infty, u(0,t) = e^t, u(x,0) = x, u_t(x,0) = 0$.
2. (20pts) Consider the following wave equation:

$$u_{tt} = u_{xx}, 0 < x < 1$$

$$u(x,0) = 1, u_t(x,0) = 1,\ 0 < x < 1$$

$$u(0,t) = u(1,t) = 0$$

Find $u(\frac{1}{2}, 3)$.

3.(20pts) Solve

$$u_t = ku_{xx} + e^x, -\infty < x < \infty$$

$$u(x,0) = x, -\infty < x < +\infty$$

4. (20pts) Solve

$$u_t = ku_{xx}, 0 < x < +\infty$$

$$u(x,0) = 0, 0 < x < +\infty$$

$$u(0,t) = 1$$

Write the solution in terms of $\int_0^{\frac{x}{4kt}} e^{-p^2} dp$.

5. (20pts) Consider the following diffusion equation

$$u_t = ku_{xx} + f(x,t), 0 < x < l, t > 0$$

$$u(x,0) = \phi(x)$$

$$u_x(0,t) - a_0 u(0,t) = g_1(t), u_x(l,t) + a_1 u(l,t) = g_2(t)$$

where

$$a_0 \geq 0, a_1 \geq 0$$

Use the energy method to show that the solution to the above problem is unique.