MATH400-201 Homework Assignment 7 (Due Date: by 6pm, April 14, 2016)

Please either hand in to my office or send it by email by 6pm of April 14, 2016. The solutions will be put on my website on April 15.

1. (a) Solve the following exterior domain problem

$$\begin{cases} u_{xx} + u_{yy} = 0, \quad \text{for } x^2 + y^2 > 1 \\ u(x,y) = x - 2y^2 \quad \text{for } x^2 + y^2 = 1 \\ u(x,y) \text{ is bounded} \end{cases}$$

Hint: use polar coordinate

(b) Prove that the solution in (a) is unique.

2. Use the method of separation of variables to solve the following PDE:

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0 \ \text{ in } D = \{(r,\theta) \mid 1 < r < 2, 0 < \theta < \frac{\pi}{4}\}$$

$$u(1,\theta) = 2\cos^2(2\theta), \ u(2,\theta) = 1$$

$$u_\theta(r,0) = 0, u_\theta(r,\frac{\pi}{4}) = 0$$

3. Find an infinite series representation (in terms of the Bessel function of order zero) for the wave equation problem

$$\begin{cases} u_{tt} = c^2(u_{rr} + \frac{1}{r}u_r), \ 0 \le r < a, t > 0 \\ u(a,t) = 0, t \ge 0 \\ u(r,0) = \phi(r), u_t(r,0) = \psi(r) \end{cases}$$

4. Solve the diffusion equation using the Bessel functions

$$\begin{cases} u_t = k(u_{rr} + \frac{1}{r}u_r + \frac{u_{\theta\theta}}{r^2}), \ 0 \le r < a, 0 \le \theta < 2\pi, t > 0 \\ u(a,\theta,t) = 0, t \ge 0, 0 \le \theta < 2\pi \\ u(r,\theta,0) = 1 - 2\cos\theta \end{cases}$$

5. Find an infinite series representation (in terms of the Bessel function) for the diffusion equation problem

$$\begin{cases} u_t = k(u_{rr} + \frac{1}{r}u_r + u_{zz}), \ 0 \le r < a, 0 < z < b, \ t > 0 \\ u_r(a,z,t) = 0, u(r,0,t) = 0, \ u(r,b,t) = 0 \\ u(r,z,0) = \phi(r,z) \end{cases}$$