Midterm Examination for MATH400

Total: 100 points

Answer All Questions. Show All Steps.

Date: March 4th, 2014

1. (30pts) Consider the following first order PDE for $u = u(x, y)$ given by

$$yu_x + xu_y = y^3 u \tag{1}$$

(i) (10points) Find the general solution to the PDE in terms of an arbitrary function.

Using the general solution given by (i) determine for (ii)-(iii) below whether (1) has a solution for each of the following data. If there is a solution, find out the solution.

(ii) (10points) $u = 1$ on $y = x$ for $1 \leq x \leq 2$

(iii) (10points) $u = y$ on $x = 1$ for $2 \leq y \leq 3$

2. (30points) Consider the traffic flow problem

$$\frac{\partial \rho}{\partial t} + \cos(\pi\rho)\frac{\partial \rho}{\partial x} = 0$$

Solve for $\rho(x, t)$ with the following initial condition

$$\rho(x, 0) = \frac{1}{4}, -\infty < x < +\infty;\ \rho(0-, t) = 1,\ t > 0;\ \rho(0+, t) = \frac{1}{3}, t > 0$$

3. (20pts) Solve the following wave equation:

$$u_{tt} = c^2 u_{xx} + \cos(ct)\cos x, -\infty < x < +\infty, t > 0$$
$$u(x, 0) = x, u_t(x, 0) = \sin x$$

4. (20pts) Solve the following diffusion equation

$$u_t = ku_{xx}, 0 < x < +\infty, t > 0$$
$$u(x, 0) = x, x > 0$$
$$u_x(0, t) = 0$$

You may write your answer in terms of the function $\int_0^{\frac{x}{\sqrt{4kt}}} e^{-p^2} dp$.