Homework Assignment 1 (Due Date: Jan. 23 2014)

1. Solve the following first order PDE and find where the solution is defined in the $x-y$ plane.

$$u_x + xyu_y = 0, u(x,1) = x^2$$

2. Solve $xu_x + xyu_y = u$ for $u = u(x,y)$ with date $u(1,y) = y^2$ for $0 \leq y \leq 1$ and find where the solution is defined in the $x-y$ plane.

3. Solve the following first order PDE and find where the solution becomes unbounded in the $x-y$ plane.

$$x^2u_x + xyu_y = u^3, \ u = 1 \ \text{ on the curve } y = x^2$$

4. Solve $u_t + t^2u_x = 4u$ for $x > 0, t > 0$ with $u(0,t) = h(t)$ and $u(x,0) = 1$.

5. Solve $xu_x + yu_y = 2$ with date $u(x,1) = x^2$ for $-\infty < x < +\infty$. Explain why we can not determine $u(x,y)$ uniquely for $y \leq 0$.

6. Let $u(x,y)$ solve the first order PDE

$$xu_x + yu_y = xu$$

(a). Find the general solutions. (b) Suppose we put $u = h(x)$ on $y = x$. Derive the condition that $h(x)$ must satisfy for a solution to exist.