

Homework Assignment 3 (Due Date: Feb 25, 2014)

1. (20pts) Use the change of variable to transform the following equations into one of the three standard form

(a) $2u_{xy} + u_{yy} = 0$, (b) $u_{xx} - 6u_{xy} + 10u_{yy} + u_y = 0$, (c) $u_{xx} - 4u_{xy} + 4u_{yy} - 2u_x + 3u_y = 0$

2.(20pts) Solve the following wave equation:

$$u_{tt} - c^2 u_{xx} = xt$$

$$u(x, 0) = x^2, u_t(x, 0) = 1 + x$$

3. (20pts) (a) State the well-posedness criteria for the following backward diffusion equation

$$\begin{cases} u_t + ku_{xx} = 0, t > 0, -\infty < x < +\infty, k > 0 \\ u(x, 0) = \phi(x) \end{cases}$$

(b) Use the function $u(x, t) = \frac{1}{n} e^{n^2 kt} \sin(nx)$ to show that the above problem is not well-posed.

4. (a) (20pts) Find the general solution formula for

$$\begin{cases} 2u_{tt} + 5u_{tx} - 3u_{xx} = 0, t > 0, -\infty < x < +\infty, \\ u(x, 0) = \phi(x), u_t(x, 0) = \psi(x), -\infty < x < +\infty \end{cases}$$

(b) (20pts) In part (a), find the solution with

$$\phi(x) = \sin x, \psi(x) = \cos x .$$