Homework Assignment 4 (Due Date: March 18, 2014)

1. (20pts) Consider the following equation

$$u_{xy} - 4yu_x = 0$$

(1) (5pts) What is the type?
(2) (15pts) Find the general solution
   Hint: write $u_x = v$.

2. (20pts) Solve $u_{tt} = c^2 u_{xx}$ for $0 < x < +\infty$, $u(0,t) = t^2$, $u(x,0) = x$, $u_t(x,0) = 0$.

3. (20pts) Consider the following wave equation:

$$u_{tt} = 4u_{xx}, 0 < x < 1$$

$$u(x,0) = 1, u_t(x,0) = 1$$
$$u(0,t) = u(1,t) = 0$$

Find $u(\frac{5}{2}, 2)$.

4. (20pts) Solve
$$u_t = ku_{xx}, 0 < x < +\infty$$
$$u(x,0) = 0, 0 < x < +\infty$$
$$u(0,t) = 1$$

   Write the solution in terms of $\int_0^{\frac{x}{4kt}} e^{-p^2} dp$.

5. (20pts) Consider the following diffusion equation

$$u_t = ku_{xx} + f(x,t), 0 < x < l, t > 0$$

$$u(x,0) = \phi(x)$$
$$u_x(0,t) - a_0 u(0,t) = g_1(t), u_x(l,t) + a_1 u(l,t) = g_2(t)$$

where
$$a_0 \geq 0, a_1 \geq 0$$

   Use the energy method to show that the solution to the above problem is unique.