

Homework Assignment 5 (Due Date: March 27, 2014)

1. (20pts) Solve the following wave equation

$$\begin{cases} u_{tt} = c^2 u_{xx}, 0 < x < l \\ u(x, 0) = 0, u_t(x, 0) = x \\ u_x(0, t) = 0, u(l, t) = 0 \end{cases} \quad (1)$$

2. (20pts) Solve the following diffusion equation with periodic boundary condition

$$\begin{cases} u_t = k u_{xx}, 0 < x < 1 \\ u(x, 0) = \cos^2 x \\ u(0, t) = u(1, t), u_x(0, t) = u_x(1, t) \end{cases} \quad (2)$$

3. (40pts). (a) (30pts) Solve

$$\begin{cases} u_t - u_{xx} = 0, 0 < x < 1, \\ u(x, 0) = \phi(x), 0 < x < 1 \\ u_x(0, t) + u(0, t) = 0, u_x(1, t) + 2u(1, t) = 0 \end{cases} \quad (3)$$

by separation of variables.

(d) (10pts) Under what conditions on $\phi(x)$, does the solution to (3) remain bounded as $t \rightarrow +\infty$?

4. (20pts) Consider the following eigenvalue problem

$$\begin{cases} X'' + \lambda X = 0, 0 < x < 1 \\ X(1) = X(0), X'(1) = 5X(0) + X'(0). \end{cases} \quad (4)$$

Show that all eigenvalues are **real**.

Hint: use the Lagrange's identity