Homework Assignment 6 (Due Date: April 8, 2014)

1. (30pts) Put the following two problems in Sturm-Liouville form, identify the weight function $w(x)$, and calculate the eigenvalues and eigenfunctions. Also what is the orthogonality relation for the eigenfunctions?

$$x^2\phi_{xx} + 5x\phi_x + \lambda\phi = 0, 1 \leq x \leq 2; \phi(1) = \phi(2) = 0$$

Hint: try $\phi(x) = x^r$

$$\phi_{xx} - 2\phi_x + \lambda\phi = 0, 0 \leq x \leq 1, \phi(0) = \phi(1) = 0$$

Hint: try $\phi(x) = e^{rx}$

2. (20pts) Use the method of separation variables to solve

$$\begin{cases} u_{tt} = u_{xx} + e^t \sin(3x), \quad 0 < x < \pi \\ u(x,0) = \sin(3x), u_t(x,0) = \sin(5x) \ 0 < x < \pi \\ u(0,t) = t, u(\pi,t) = 0 \end{cases} \tag{1}$$

3. (30pts) (a) (20pts) Use the method of separation variables to solve the following PDE:

$$u_{xx} + u_{yy} = 0 \ \text{ in } D = (0,\pi) \times (0,\pi)$$

$$u_y(x,0) = u(x,\pi) = 0, u(\pi,y) = 0$$

$$u(0,y) = \cos^2(y)$$

(b) (10pts) Prove that the solution obtained in (a) is unique.

4. (20pts) Use the method of separation of variables to solve the following PDE:

$$u_{xx} + u_{yy} = 1 \ \text{ in } D = \{(x,y)|x^2 + y^2 < 4\}$$

$$u(x,y) = x^2 - y^2 \ \text{ on } \partial D = \{(x,y)|x^2 + y^2 = 4\}$$