

# MATH 516 INTRODUCTION TO PARTIAL DIFFERENTIAL EQUATIONS (I)

*Term 1 (Sept-Dec 2015)*

<http://www.math.ubc.ca/~jcwei>

**Instructor:** Juncheng Wei, LSK 303B, Tel. 604-822-6510, E-mail: [jcwei@math.ubc.ca](mailto:jcwei@math.ubc.ca)

**Content:** This course is an introduction to the qualitative theory of partial differential equations (PDEs). It should be useful to students with interests in applied mathematics, differential geometry, mathematical physics, probability, harmonic analysis, dynamical systems, and other areas, as well as to PDE/Analysis-focused students. We will review and expose a few analytic tools along the way, e.g. Fourier transform and weak convergence.

**Prerequisites:** Basic real analysis, including convergence, Lebesgue integral and  $L^p$  spaces.

## Topics:

- 1. Classical linear equations
  - (a) Laplace, heat, and wave equations; their solution formulas
  - (b) mean value properties and maximum principles, applications to uniqueness and regularity
  - (c) existence: Perrons subsolution method and Dirichlet principle
  - (d) regularity of weakly harmonic functions, analyticity, Harnack inequality, etc
- 2. Classical solutions of second order elliptic equations (4 hours)
  - (a) weak and strong maximal principles
  - (b) Holder spaces and Schauders a priori estimates
  - (c) existence by the method of continuity
- 3. Sobolev spaces
  - (a) weak derivatives and Sobolev spaces

- (b) inequalities of Sobolev, Morrey, Poincare, and Gagliardo-Nirenberg
- (c) approximations, extensions, trace, compactness, and dual spaces
- 4. Weak solutions of elliptic equations in divergence form
  - (a) weak solutions and maximal principle
  - (b) existence and eigenvalues by Lax-Milgram theorem and Fredholm alternative
  - (c) regularity
  - (d) application to semilinear elliptic problems
  - (e) analogous results for 2nd-order parabolic equations
- 5. Variational and non-variational techniques
  - (a) Direct method of calculus of variations
  - (b) Constraint methods
  - (c) Mountain-pass theorem
  - (d) Fixed point methods
  - (e) Method of sub-super solutions
  - (f) Monotonicity and Pohozaev identities

**References:** We will mainly follow Evans book and also use materials from the others.

1. Partial Differential Equations, 2nd ed., by L. C. Evans, American Math Society, 2010. See authors homepage <http://math.berkeley.edu/~evans> for errata. This is a general text suitable for a first course and also for reference.

2. Partial Differential Equations, 4th ed., by Fritz John, Springer-Verlag. This is a classic textbook and contains materials not found elsewhere, e.g. Weyls lemma and extended treatise on wave equation.

3. Elliptic Partial Differential Equations of Second Order, 2nd ed., by David Gilbarg and Neil S. Trudinger, Springer-Verlag, Classics in Mathematics series. This book is specialized in elliptic equations and is a standard reference.

4. Elliptic Partial Differential Equations, by Qing Han and Fanghua Lin, Courant lecture Notes. This book is easy to read and cover most of the materials in book 3.

In each week, I will point to the exact chapter of the book.

**Assessment:** The grade is based on homework assignments (five or six).