Method of Undetermined Coefficients

$$ay'' + by' + cy = g(t)$$

$g(t)$	$y_p(t)$
Polynomial: $a_0 t^n + a_1 t^{n-1} + \dots + a_n$	$t^s (A_0 t^n + A_1 t^{n-1} + \dots + A_n)$
$e^{\alpha t}(a_0 t^n + a_1 t^{n-1} + \dots + a_n)$	$t^s e^{\alpha t}(A_0 t^n + A_1 t^{n-1} + \dots + A_n)$
$\cos\beta t$ or $\sin\beta t$	$t^s (A\cos\beta t + B\sin\beta t)$
$e^{\alpha t}\cos\beta t$ or $e^{\alpha t}\sin\beta t$	$t^s e^{\alpha t}(A\cos\beta t + B\sin\beta t)$

Here $s = 0$ or 1, or 2 is the least integer such that y_p does not contain solutions of homogeneous equation.

Ex. 1 $\quad y'' + 2y' + y = 3e^{2t}$, $\quad y_1 = e^{-t}$, $y_2 = te^{-t}$

$y_p = t^s e^{2t} \Rightarrow s = 0$

Ex. 2 $\quad y'' + 2y' + y = 3e^{-t}$, $\quad y_1 = e^{-t}$, $y_2 = te^{-t}$

$y_p = t^s e^{-t} \Rightarrow s = 2$

Ex. 3 $\quad y'' + 2y' + y = 3te^{-t}$, $\quad y_1 = e^{-t}$, $y_2 = te^{-t}$

$y_p = t^s (A_0 t + A_1)e^{-t}$, $\quad s = 2$

Ex. 4 $\quad y'' + 2y' + y = e^{-t}\cos t$, $\quad y_1 = e^{-t}$, $y_2 = te^{-t}$

$y_p = t^s e^{-t}(A\cos t + B\sin t) \Rightarrow s = 0$

Ex. 5. $\quad y'' + y = 3\cos t \qquad y_1 = \cos t$, $y_2 = \sin t$

$y_p = t^s (A\cos t + B\sin t) \Rightarrow s = 1$