

## CORRIGENDUM

# A model of tear-film breakup with continuous mucin concentration and viscosity profiles – CORRIGENDUM

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Two errors were made in evaluating the dimensional parameters of table 1 and in converting them to the dimensionless parameters in Dey *et al.* (2019). This led to an incorrect time scale in computing the dimensional tear-film breakup time in figure 12. All the other results, presented in figures 3–11, are dimensionless and remain correct for the dimensionless parameters employed therein.

To rectify the errors, we first reinstate a factor of  $6\pi$  in the Hamaker constant omitted by mistake; the correct value is  $\mathcal{A} = 6\pi \times 3.5 \times 10^{-19}$  J m (Braun *et al.* 2018). Besides, we revise the tear-film thickness to  $H = 0.5$   $\mu\text{m}$ . The new  $H$  value is thinner than the experimentally reported tear-film thickness by a factor of 7–10. This is based on recent findings that the tear film undergoes a rapid initial thinning owing to factors such as evaporation (Braun *et al.* 2018), lipid clustering on the interface (Zhong *et al.* 2019) and lid-associated thinning, including drainage (King-Smith, Begley & Braun 2018). Thus, within several seconds, the tear film thins from a thickness of about 3.5  $\mu\text{m}$  to 0.5  $\mu\text{m}$  (Braun *et al.* 2018). It is only after this initial phase that the van der Waals force becomes the dominant driving force for tear-film rupture. As our continuous viscosity model (CVM) focuses solely on the van der Waals-driven rupture and ignores the initial rapid thinning, we should adopt the  $H$  value at the end of the rapid thinning. The changes in  $\mathcal{A}$  and  $H$  lead to new values for four dimensionless parameters:  $\mathcal{C} = 0.03$ ,  $Pe_s = 0.02$ ,  $\Delta_b = 10^{-4}$  and  $\mathcal{M} = 0.05$ . With the corrected parameters and time scale, figure 1 plots the dimensional rupture time  $t_{rup}$  as a function of  $\mathcal{C}$ . This replaces the original figure 12.

The new  $t_{rup}$  turns out to be quite close to that reported in the original figure 12. For a reasonable range of the interfacial tension,  $\mathcal{C}$  now lies in the range of  $10^{-3}$ –0.1, over which  $t_{rup}$  varies from 2.4 to 135 s. For the baseline value  $\mathcal{C} = 0.03$ , the rupture time is  $t_{rup} = 44.4$  s. Hence, the previous conclusion – drawn from comparisons with experimental data and two-layer model predictions – stands: the CVM is superior in reproducing experimental measurements of healthy-eye tear film breakup time.

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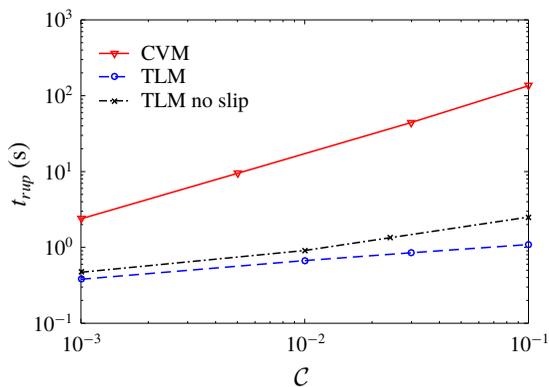


FIGURE 1. Dimensional rupture time  $t_{rup}$  as a function of the dimensionless surface tension  $C$ . The dotted lines are predictions of the two-layer model (TLM) with and without slip on the substrate. This replaces the original figure 12 in Dey *et al.* (2019).

### Declaration of interests

The authors report no conflict of interest.

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