

# Anomalous Rolling of Spheres Down an Inclined Plane

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## **Abstract**

A sphere in air will roll down a plane which is tilted away from the vertical. The only couple acting about the point of contact between the sphere and the plane is due to the component of the weight of the sphere along the plane, provided that air friction is negligible. If on the other hand the sphere is immersed in a liquid, hydrodynamic forces will enter into the couples, which turn the sphere, and the rotation of the sphere can be anomalous, i.e., as if rolling up the plane whilst it falls. In this paper we shall show that anomalous rolling is a characteristic phenomenon which can be observed in every viscoelastic liquid tested so far. Anomalous rolling is normal for hydrodynamically-levitated spheres, both in Newtonian and viscoelastic liquids. Normal and anomalous rolling are different names for dry and hydrodynamic rolling. Spheres dropped at a vertical wall in Newtonian liquids are forced into anomalous rotation and are pushed away from the wall whilst in viscoelastic liquids, they are forced into anomalous rotation, but are pushed towards the wall. If the wall is inclined and the fluid is Newtonian, the spheres will rotate normally for dry rolling, but the same spheres rotate anomalously in viscoelastic liquid when the angle of inclination from the vertical is less than some critical value. The hydrodynamic mechanisms underway in the settling of circular particles in a Newtonian fluid at a vertical wall are revealed by an exact numerical simulation based on a finite-element solution of the Navier-Stokes equations and Newton's equations of motion for a rigid body.

## 1. Introduction

Goldman, Cox and Brenner [1967] treat the problem of interaction between a sphere and a wall. They also consider the problem of a sphere ‘rolling’ down an inclined wall and find that the sphere cannot be in physical contact with the wall and that it slips, giving rise to anomalous rotation when forced into close approach. In this paper ‘anomalous’ is defined as the sense of rotation that exists when the sphere rotates as if it were rolling up the wall. We define ‘normal’ rotation as the sense of rotation that exists when the sphere rotates as if it were rolling down the wall. Bungay and Brenner [1973] showed that the rotation of a tightly-fitting ball falling down a vertical tube would change sense, from normal to anomalous, as the distance between the ball and the tube wall tended to zero. The Stokes flow predictions of these authors involve neither inertia nor elasticity. The phenomenon of anomalous rolling predicted by these authors appears in the experiments of Humphrey and Murato [1991]. They found that the rotation of a sphere gradually changes from anomalous to normal as the tube inclination angle is increased and the sphere contacts the wall. They think that inertia-induced lift keeps the rolling ball off the wall at the smaller angles of inclination.

Dhahir and Walters [1989] studied flow past a cylinder in a straight channel of rectangular cross section. They did experiments with Newtonian fluids, 2% and 3% Xanthan (Kelco) in water, and 1.5% aqueous polyacrylamide (an elastic fluid). Looking at a horizontal Poiseuille flow moving from left to right, with a cylinder above the center line, the cylinder will rotate against the clock as if turned by shears from the center part of the channel rather than from the gap between the cylinder and the wall. For all of the non-Newtonian fluids, the flow generated a side force which pushed the cylinder toward the wall, no matter where the cylinder was placed. This side force was too small to measure in Newtonian liquids. These results are totally consistent with phenomena we have observed on the lateral motions of spheres settling along a wall.

Jones and Walters [1989] did experiments on the flow of polymeric liquids through a channel blocked by a periodic array of staggered cylinders, simulating a porous media. They found that the elastic polyacrylamide did not want to pass through the narrow passages between the channel wall and

the cylinder, but the Newtonian liquids and aqueous Xanthan were not blocked. Blockage, even partial blockage, can produce a situation in which a cylinder or sphere is turned by the shears from the fluid which goes around the outside of a sphere, leading to the rolling which we have called anomalous, but which is normal when the turning is controlled by hydrodynamics. A mechanistic description of the hydrodynamics underway for the rotations induced by the settling of a circular particle in two dimensions will be presented in §5, but our understanding of the fluid mechanics underway in the viscoelastic case has yet to be revealed.

### Figure 1

Joseph, Nelson, Hu and Liu [1992] found anomalous rolling of a sphere along an inclined wall. The angle between the wall and gravity was varied so that the sphere fell on, rather than away from, the wall. A sphere falling down these inclined walls rotated normally in viscous liquids as it does in dry rolling, but rotated anomalously in the other sense in viscoelastic liquids when the wall was not tilted too far from the vertical. In this paper we document this phenomenon by reporting observations and measured data for many different polymeric liquids. We find anomalous rolling in all liquids, Newtonian and viscoelastic, when the wall is vertical even though spheres are repelled by the wall in Newtonian liquids and attracted to the wall in viscoelastic liquids. The anomalous results for rolling spheres in apparent (only) contact with tilted walls are as reported by Joseph, et al. [1992] with the

caveat that viscoelastic solutions with weak or no normal stresses stand between Newtonian and viscoelastic behavior.

## 2. Material and dimensionless parameters

The material parameters (Table 1) which were measured in the liquids used in the experiments are the density  $\rho$ , viscosity  $\eta = k\dot{\gamma}^{n-1}$  where  $\dot{\gamma}$  is the shear rate in reciprocal seconds, the climbing constant  $\hat{\beta}$  measured on a rotating rod viscometer (Beavers and Joseph [1975]) and the wave speed  $c$ . To compute  $\hat{\beta}$  from measured values of the climb, we need the interfacial tension which we measured with a spinning drop tensiometer (Joseph, *et al* [1992]). The value of  $\hat{\beta}$  is insensitive to a small change of surface tension (chapter 16 in [J1990]).

The climbing constant  $\hat{\beta}$  is related to the limiting (zero shear) value of the first and second normal stress differences

$$(n_1, n_2) = \lim_{\dot{\gamma} \rightarrow 0} (N_1(\dot{\gamma}), N_2(\dot{\gamma})) / \dot{\gamma}^2 \quad (2.1)$$

by

$$\hat{\beta} = \frac{1}{2} n_1 + n_2 \quad (2.2)$$

The climbing constant

$$\hat{\beta} = 3\alpha_1 + 2\alpha_2 \quad (2.3)$$

may also be expressed in terms of quadratic constants

$$(\alpha_1, \alpha_2) = \left(-\frac{1}{2} n_1, n_1 + n_2\right) \quad (2.4)$$

of the second order fluid.  $\alpha_2 / |\alpha_1|$  is the ratio of quadratic constants and

$$[\alpha_1, \alpha_2] = [-m, 2m - 2] \hat{\beta} / (m - 4) \quad (2.5)$$

where  $m = 2\alpha_1/(2\alpha_1 + \alpha_2) = -n_1/n_2$  is the ratio of the first to second normal stress difference. It can be argued (§17.11 in [J1990]) that  $m = 10$  is a reasonable value for our polymer solutions. Then

$$\frac{\alpha_2}{|\alpha_1|} = \left| \frac{2(1-m)}{m} \right| = 1.8 \quad (2.6)$$

is a constant and  $\alpha_1$  and  $\alpha_2$  are determined by the measured values of the climbing constant  $\hat{\beta}$ . The value of  $n_1$  we get from measuring  $\hat{\beta}$  is not sensitive to the value of the ratio  $n_2/n_1$  as long as  $n_2$  is relatively small and negative (see §17.11 in [J1990]).

The measured value of the climbing constant together with the assumption that the second normal stress difference is -1/10 as large as the first, allows us to evaluate Roscoe's [1965] formula

$$T_{11} - T_{22} = 3\dot{\gamma}\eta_o + 3(\alpha_1 + \alpha_2)\dot{\gamma}^2 \quad (2.7)$$

for the extensional stress difference where  $\dot{\gamma}$  is the rate of stretching in the direction  $x_1$  and  $\eta_o$  is the zero shear viscosity. Using (2.6) and  $\alpha_1 = -n_1/2$  we get

$$T_{11} - T_{22} = 3\dot{\gamma}\eta_o + 1.2n_1\dot{\gamma}^2 \quad (2.8)$$

The zero shear value of the first normal stress difference  $n_1 = \frac{2m}{m-4}\hat{\beta} = \frac{10}{3}\hat{\beta}$  and the zero shear quadratic correction  $4\dot{\gamma}\hat{\beta}$  of Troutons viscosity,  $3\eta_o$ , increase with  $\hat{\beta}$ . An argument given by Liu and Joseph [1993] suggests that extensional stresses, broadly speaking, control the properties of the aggregation of particles in viscoelastic liquids documented here.

Fluid	$\rho$ (gm/cm <sup>3</sup> )	$\eta_o$ (Pa.sec)	$k$	$n$	$\hat{\beta}$	$n_1$ (gm/sec)	$c$ (cm/sec)	$\lambda$ (sec)
1.5% aqueous polyox	1.00	101	10.1	0.38	132	440	20.3	0.245
1.25% aqueous polyox	1.00	55.1	6.42	0.39	117	389	17.2	0.186
1.0% aqueous polyox	1.00	31.8	3.97	0.42	108	360	15.0	0.162
0.4% Carbopol in 50/50 glycerin/water	1.13	0.76	0.31	0.67	0	0	15.9	0.0027
0.3% aqueous Xanthan	1.00	5.21	1.1	0.28	0	0	12.2	0.035
S1	0.875	8.06	7.14	0.62	11.8	39.3	72.4	0.0018
STP	0.86	18.0	17.8	0.85	0.97	3.23	2.86	0.00026

Table 1. Material parameters of fluids used in experiments. Percentages are by weight. Experiments were also done in more dilute polyox solutions (WSR 301) and in a 1.2% polyacrilamide solution in a 50/50 glycerin-water solution, but the material parameters were not measured. The numerical value  $k$  is the value of the viscosity in Pa.sec at a shear rate  $\dot{\gamma} = 1$

Glycerin and water mixtures in various concentrations were used to determine Newtonian behavior. The polyox and polyacrilamide solutions are standard test viscoelastic liquids exhibiting normal stresses, shear thinning and memory effects.

STP is a solution of polysulutilene in petroleum oil which was used extensively in early studies of rod climbing (see Joseph, [1990]). S1 is a solution of 50.0g of 5% W/W of PBI in decalin plus 50.0g polybutene oil. It is a world-wide standard test fluid which is being characterized by different laboratories in many countries. We mixed our own samples according to procedures laid down by Professor J. Ferguson of the University of Glasgow. Our homemade solutions have nearly the same properties as the premixed samples given to us.

The viscosity of these two polymer solutions was measured as a function of the shear rate  $\dot{\gamma}$  on the RSF2 Rheometrics fluid rheometer and is given in Figure B. The viscosity of STP is nearly constant for shear rates less than 100. The viscosity of S1 decreases with increasing  $\dot{\gamma}$ , but the decrease is very slow for shear rates less than 10. The viscosity of S1 is an order of magnitude smaller than STP; it is a much more mobile liquid. Both solutions climb a rotating rod, but the STP is not a good climber; the climbing constant at room temperature is about one gm/cm. We can say that STP is a Boger fluid

with very weak normal stresses. The climbing constant of S1 at 25°C is approximately 41 gm/cm and S1 can be said to resemble STP with much larger normal stresses, especially at low rates of shear.

Values for the dynamic moduli of STP and S1 are given in Figure C. The loss modulus for STP is an order of magnitude higher than S1. The storage modulus of S1 is larger than STP for shear rates less than about 10 s<sup>-1</sup>, and the shear rate at which the loss modulus falls below the storage modulus is much lower in S1 than in STP. S1 is a more mobile and much more elastic liquid than STP.

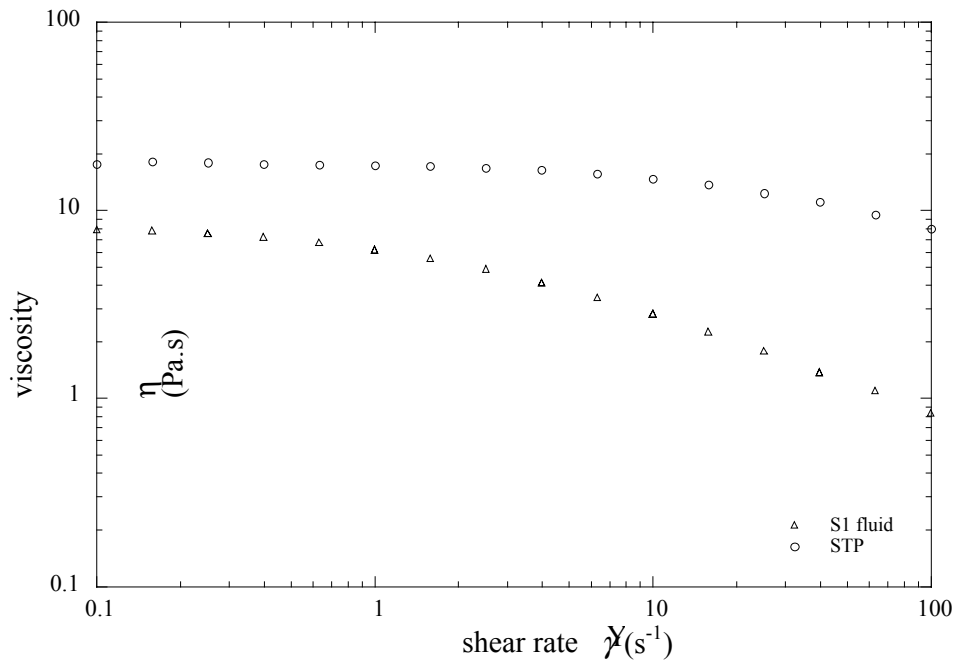


Figure B. The viscosities of STP and S1 fluid as a function of the shear rate were measured, at temperature of 24.5°C, on a RSF2 Rheometrics fluid rheometer with a cone of 50 mm diameter and 0.021 radian cone angle and a plate. The viscosity of S1 is an order of magnitude smaller than STP; it is a much more mobile liquid. S1 has a small shear thinning, and STP is more like a Boger fluid. The power law constants for S1, for the shear rates greater than 0.5 s<sup>-1</sup>, are  $\kappa=7.14$  and  $n=0.62$ ; whereas for STP,  $\kappa=18.7$ ,  $n=0.85$ . And for S1, the density is  $\rho=0.875$  gm/cm<sup>3</sup>, the zero shear viscosity is  $\eta_0=8.06$  Pa.s; whereas for STP,  $\rho=0.86$  gm/cm<sup>3</sup> and  $\eta_0=18.0$  Pa.s.



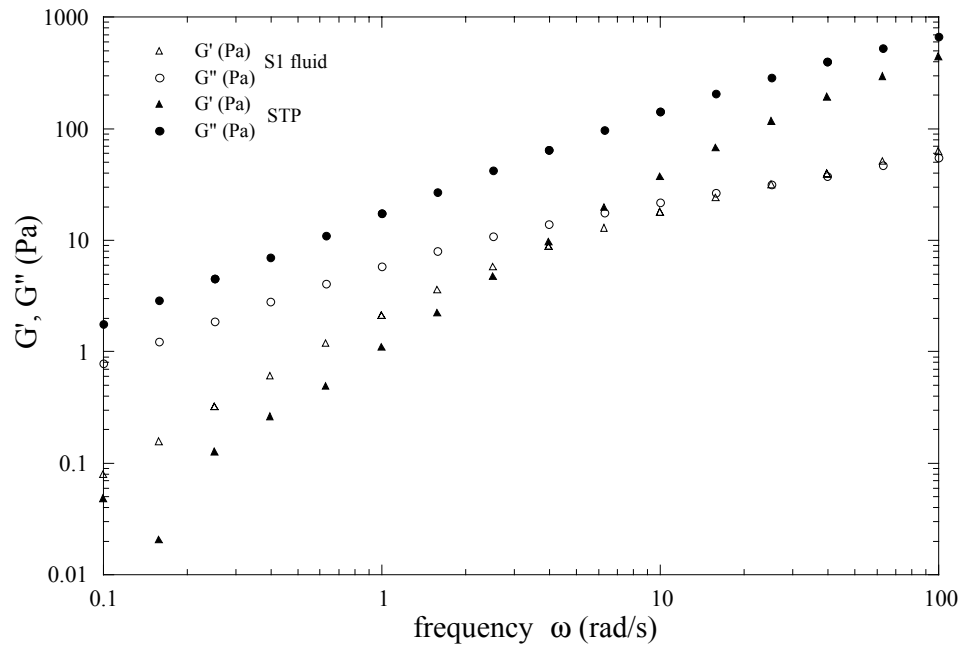


Figure C. Dynamic moduli of STP and S1 were measured, at 2% strain, on the same rheometer with the same pair of cone and plate as in Figure 2. The loss modulus for STP is an order of magnitude higher than S1. The storage modulus of S1 is larger than STP for shear rates less than about 10. The shear rate at which the loss modulus falls below the storage modulus for S1 is much lower than that for STP.

We attempted to isolate the role of shear thinning suppressing both normal stresses and elasticity by using a solution of 0.4% Carbopol 690 (Goodrich) in a 50/50 glycerin/water mixture in our sedimentation experiments. The viscosity versus shear rate for this Carbopol solution is plotted in Figure D, and the dynamic moduli are plotted in Figure E. Carbopol is thought to be a pseudoplastic fluid without elasticity. Since our Carbopol solution has a non-zero storage modulus, it cannot be said to be without elasticity. The presence of small elasticity in Carbopol solutions has been noted before; for example, Hartnett and Kostic [1989] have noted that an aqueous solution of 1000 wppm Carbopol exhibits a phase shift between the input shear stress response of less than  $\pi / 2$  radians, and that there is evidence that aqueous Carbopol solutions experience strong secondary motions in laminar flow in noncircular channels, but they do not reduce drag. There is no evidence that Carbopol 690 in 50/50

glycerin/water has a measurable value of the first normal stress difference, and it does not climb a rotating rod.

To determine the effects of shear thinning with a strong memory but no normal stresses, we used a solution of 0.3% Xanthan (Kelco) in water. The graph of viscosity versus shear rate is shown in Figure D, and the variation of the storage and loss moduli with frequency is shown in Figure E. This Xanthan solution is very shear thinning and it apparently has no normal stresses. We could not register a first normal stress difference on the Rheometrics fluid rheometer and the 0.3% Xanthan solution would not climb a rotating rod. On the other hand, this fluid has a high storage modulus and can be said to be linearly elastic.

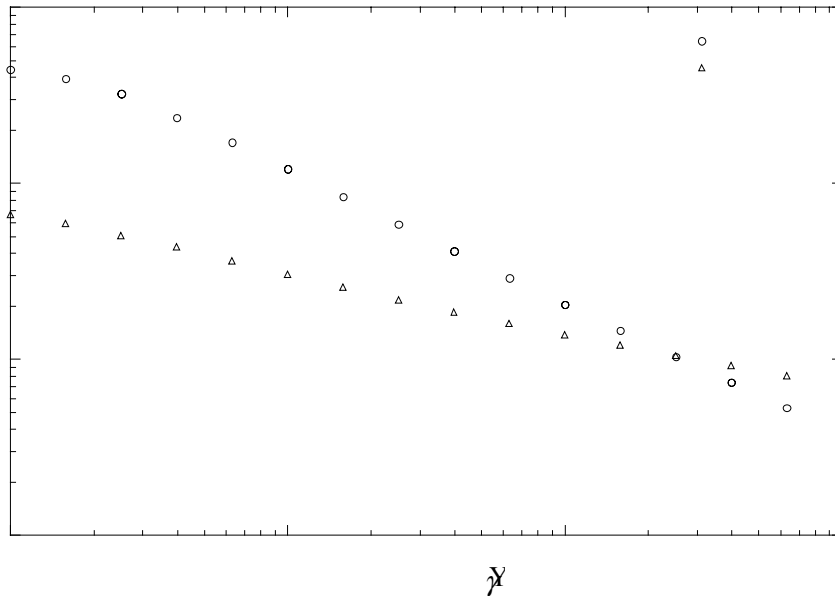


Figure D. The viscosities of 0.3% aqueous Xanthan and 0.4% Carbopol in 50/50 glycerin/water solution as a function of the shear rate at temperature of 24.5°C. The Xanthan solution has a higher but more shear thinning viscosity than the Carbopol solution. For the Xanthan solution,  $\kappa=1.1$ ,  $n=0.28$ ,  $\rho=1.0 \text{ gm/cm}^3$ ,  $\eta_0=5.21 \text{ Pa.s}$ ; whereas for the Carbopol,  $\kappa=0.31$ ,  $n=0.67$ ,  $\rho=1.13 \text{ gm/cm}^3$ , and  $\eta_0=0.76 \text{ Pa.s}$ .

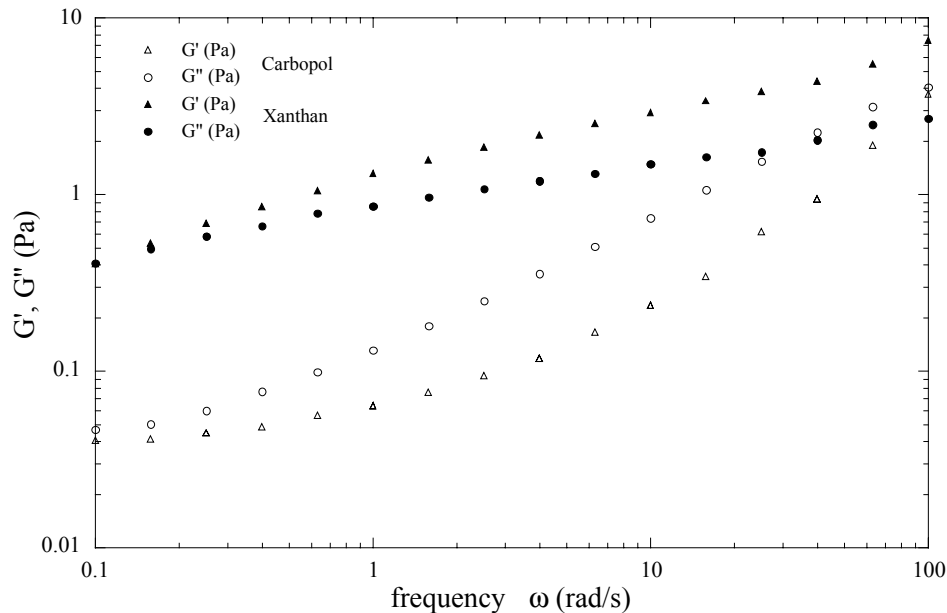


Figure E. Dynamic moduli of Xanthan and Carbopol solutions. In modest shear rates ranging from 0.1 to 100, both the storage modulus and loss modulus of the Xanthan solution are higher than that of the Carbopol solution. And also, in the Xanthan, the storage modulus  $G'$  is greater than the loss modulus  $G''$ , but in the Carbopol it is the opposite.

### 3. Rolling of spheres down the sidewall of a channel. Experimental Setup

Six different types of spheres and three channels were used in our experiments which are shown in Table 2. The motion of 1/4-inch diameter sedimenting spheres in the thin bed is basically two dimensional, with spheres centering themselves between two close walls. The centering was described by Liu and Joseph [1993]. Close sidewalls have a marked effect on the magnitude of the fall velocity and rotation rate of spheres rolling down an inclined wall. But the direction of the rolling and other qualitative properties of aggregation of sedimenting spheres, sphere-wall interactions and the tilting of long sedimenting bodies, do not depend strongly on the aspect ratio.

spheres	material	plastic	teflon	rubber	steel	steel	tungsten
	diameter (inches)	0.25	0.25	0.25	0.25	0.5	0.25
	density (g/cm <sup>3</sup> )	1.34	2.18	5	7.61	7.61	15.8
channel dimensions (inches)	0.275x4x23		glycerin		polyox polyacrylamide STP glycerin		STP
	0.85x0.85x18	Xanthan Carbopol glycerin	Xanthan Carbopol S1 glycerin	Xanthan	S1 STP glycerin	STP	S1
	1x1.63x28					STP	

Table 2. The channels and the spheres used in our experiments. The cells in the right-bottom corner of the table give the liquids we tested with certain spheres in certain channels.

In this experiment, we tilted our sedimentation channel with its center plane vertical and the side walls inclined to the vertical. The angle of inclination from the horizontal is  $\theta$  (Figure F). When  $\theta = 90^\circ$ , the plane of rolling is vertical. We measured the fall velocity and rolling velocity of spheres on a Kodak Spin Physics. The measurements were routine and accurate. We were unable to get an accurate measurement of the distance between the wall and the sphere in the viscoelastic case of close approach. The drag on the sphere must be strongly influenced by the precise value of this too-small-to-measure stand-away distance. A more systematic study than the one undertaken here would look systematically at weight and size effects of particles. We did many casual tests of this type, but found no surprising results. Our view is that more systematic experiments should be undertaken only after some hypotheses about the controlling viscoelastic mechanisms have been formulated for testing.

The case of vertical settling ( $\theta = 90^\circ$ ) is special. If a sphere is initially dropped at or near a vertical wall, it will always turn counter-clockwise, as shown in Figure F(b), whether the fluid is viscoelastic or inelastic. We have found that spheres dropped in a viscoelastic liquid near a vertical wall are sucked to the wall, but the same spheres will move a certain distance away from the wall when they are dropped in inelastic fluids. This striking difference will be documented in another paper (Joseph and Liu [1993b]). For now it will suffice to note once again that independent of whether the sphere migrates to the wall or takes up a small stand-off as it falls, it will turn counter-clockwise as it falls.

#### **4. Rolling of spheres down the sidewall of a channel. Experimental results.**

We might have thought that friction emanating from the wall would turn the sphere clockwise. Evidently the small gap between the falling sphere and the wall partially blocks the fluid, so that the main flow and the main shears are on the outside of the sphere, where passage is not blocked, turning the sphere, as in Figure A. The sense of the rotation of the sphere, then, which is dropped from rest in close proximity to a vertical wall, must be anomalous, since much of the fluid cannot get through the small gap between the wall and the sphere, and instead must go around the outside. The blockage effect is greatly enhanced in viscoelastic fluids because the viscoelastic forces draw the spheres to the wall, even when the wall is vertical.

It is evident from the considerations just introduced that the fluid dynamics of sphere-wall interactions are such as to produce anomalous rolling when the relative weight of the sphere is not sufficient to hold it on the wall against countervailing hydrodynamics lift forces. This is always the case when the wall is vertical and even when  $\theta$  is very small. In this sense, anomalous rolling is normal whenever hydrodynamic lift forces are at work. The lift forces are sensitive to the Reynolds number and a sphere will “fly” if the forward speed is fast enough no matter what the value of the angle of tilt. Spheres settling on an inclined wall however need not reach the forward velocity necessary for levitation against the component of the buoyant weight of the sphere pushing it on the wall.

Our experiments using the 0.275 x 4 x 23 inch channel filled with pure glycerin and steel balls are representative for Newtonian liquids. When  $\theta = 0^\circ$ , vertical settling, a sphere started at the wall, will be repelled by the wall and will commence to roll in the anomalous sense as in Figure F(b). When  $\theta \leq 89^\circ$ , the sphere falls to the wall and a gap is not evident; it either slips at the wall ( $11^\circ \leq \theta \leq 14^\circ$ ) or slips a lot and rolls normally a little ( $14^\circ \leq \theta \leq 20^\circ$ ), rotates normally with a little slip ( $20^\circ \leq \theta \leq 40^\circ$ ) or rotates normally with no slip ( $\theta \geq 40^\circ$ ).

The interval of  $\theta$  for normal rolling and slipping are functions of the Reynolds number and hence change with the weight of the sphere and the aspect ratio of the channel. We carried out experiments with Teflon spheres in the narrow channel and steel and Teflon spheres in the square channel with different quantitative but the same qualitative results. We have already remarked that we expect the spheres to lift off the inclined walls at a higher Reynolds number, not seen in our experiments.

Our experiments in 0.4% Carbopol in 50/50 glycerin-water solutions were carried in the 0.85 x 0.85 x 18 inch channel using plastic and Teflon balls. The wall does not attract these balls when it is vertical ( $\theta = 0^\circ$ ); a sphere dropped next to the wall will rotate in the anomalous way and drift away from the wall to an equilibrium stand-away distance, as in Newtonian fluids. In the case of tilted walls, with slight tilting ( $70^\circ \leq \theta \leq 80^\circ$ ) the sphere falls to the wall, first rotating up the wall, then slipping without rolling. This behavior is intermediate between Newtonian and viscoelastic behavior, but is more Newtonian.

We turn next to the experiments with viscoelastic liquids in which a vertical wall attracts rather than repels sedimenting spheres; that is, all solutions mentioned in this paper other than glycerin and Carbopol. Naturally, an inclined wall will attract a sphere more strongly if a vertical wall attracts. The anomaly is that in these cases of sedimentation with close approach, as if touching, the sphere rotates anonymously, with “dry” rolling taking over only for relatively horizontal tilting of the wall.

In Table 3 we give the measured values of the terminal fall velocity  $U_0$  and the terminal angular velocity  $\omega_0$ . It is of interest to compare the measured values of  $U_0$  with computed values of the Stokes velocity  $U_s$ , which is generated from the balance between buoyancy and drag  $\Delta\rho g \frac{4}{3} \pi a^3 = 6\pi\eta(1)aU_s$ , where  $\Delta\rho$  is the density between the solid (Table 2) and fluid (Table 1),  $g = 980$  cm/sec,  $a = 1/4$  inch or  $1/2$  inch is the sphere radius and  $\eta(1)$  ( $=k$ ) is the viscosity at  $\dot{\gamma} = 1$  (Table 1), which we used because of uncertainty in the value of  $\eta_0$ . In all cases, the Stokes drag is much smaller than the drag in our experiments. The main reason for this discrepancy is that the nearby wall to which the sphere is attracted exerts an additional drag. Nearby side walls also exert an additional drag. The drag on the  $1/2$ -inch sphere in the  $1 \times 1.63$ -inch channel is very strongly influenced by the side walls. The measured value of  $\omega_0$  is corrected with the measured value of  $U_0$  by the following argument. In dry rolling  $U_0 = a\omega_0$  so that in hydrodynamic rolling, we might find a relationship like  $U_0 = ca\omega_0$  with an unknown  $c$  between zero and one. Of course the sense of  $\omega_0$  is reversed in hydrodynamic rolling, and the shear from the outside is a less-effective turner than dry friction.

Table 3

Our experiments in 0.3% Xanthan in water were carried out in the square channel, using plastic, Teflon and rubber balls. The Xanthan solution is very interesting because it has an appreciable linear elasticity (a high storage modulus) and is very shear thinning (Table 1), yet has no measurable normal

stress. A plastic sphere, which is light and settles slowly ( $U_0 = .269$  cm/sec) is attracted to a vertical wall where it rotates anomalously with a slow angular velocity ( $\omega_0 = 0.126$  rad/sec). The heavier Teflon sphere falls faster ( $U_0 = 13.12$ ) than the shear wave speed ( $c = 12.2$ ) and it levitates off the wall. The fall velocity for Teflon and rubber spheres is too great to determine the value of the angular velocity. In the case of a tilted wall, data was taken only for Teflon spheres. These were attracted to the wall when  $\theta \leq 85^\circ$  even though the wall repels the sphere at  $\theta = 0^\circ$ . The rotation was very weak, with appreciable amounts of slipping for angles of  $15^\circ$ ,  $20^\circ$ ,  $25^\circ$  and  $30^\circ$ . The weak rolling was anomalous for  $20^\circ$  and  $25^\circ$ , there was only slip at  $\theta = 25^\circ$  and slip plus normal rolling at  $\theta = 30^\circ$ . The plastic sphere was attracted to the vertical and inclined wall where its rotation was too slow to measure. The rubber sphere to the wall at  $\theta \geq 15^\circ$  where it settled so fast that sense of rotation could not be ascertained. It is certain that a lighter sphere would levitate at the large velocities at which the rubber spheres fell. The behavior of spheres rolling on walls in Xanthan is intermediate between Newtonian and viscoelastic, but is more viscoelastic.

In our experiments using 1/4-inch steel balls on the flat side wall of the narrow channel (Table 2) filled with 1.2% polyacrilamide in 50/50 aqueous-glycerin, we found intermittent slipping interspersed with anomalous rolling for  $10^\circ < \theta \leq 30^\circ$ . Only slip was observed for  $35^\circ$  and  $40^\circ$  and intermittent slipping and normal rolling for  $\theta = 35^\circ$ ,  $40^\circ$  and  $44^\circ$ . In all cases a very slight gap could be detected between the ball and the plane; the ball was hydrodynamically levitated in a position of close approach.

We did the same experiment with a 1/4-inch wire laid against the flat surface. The contact between the sphere and the wire is less severe than the contact between the sphere and a flat wall. The gap under rolling on the inclined wire changes radically across the gap. There are only very small differences in the outcome of experiments using these two wall surfaces.

Now we give some measured results in graphical form for the terminal settling and angular speed of rolling spheres as a function of the angle  $\theta$  of inclination of a tilted wall in aqueous Polyox,



STP and S1. The settling speed  $U$  and angular speed  $\omega$  are normalized by their values  $U_0$  and  $\omega_0$  at  $\theta = 0^\circ$ . In all cases, anomalous rolling is observed.

We have presented the data in two forms. First we give  $U/U_0$  as a function of  $\sin \theta$ . The force driving the tangential motion of the sphere down the plane is the buoyant force  $\Delta\rho g \sin \theta \frac{4}{3} \pi a^3$ , linear in  $\sin \theta$ . For a linear response between the velocity and driving force and velocity, we might expect that  $U = c_1 \sin \theta$  where  $c_1$  depends on geometrical and fluid parameters and is a sort of drag coefficient. We have attempted to make a best guess at the parameter from the measured data, but the response is not rigorously linear and the fits are not convincing. In fact we really don't expect linearity for no other reason than that the normal force exerted on the inclined plane varies with  $\theta$ . This variation will produce a change in the small gap between the sphere and the plane and change the drag emanating from the wall.

A second form for presenting data is as  $\omega/\omega_0$  versus  $U/U_0$ . We expect to see a more or less linear relation  $U/U_0 = c_2 \omega/\omega_0$  based on the idea that  $\omega$  and  $U$  have the same dependence on  $\sin \theta$ , whatever that dependence might be. This idea seems to work

In Figures H and I, we have presented results for aqueous Polyox (WSR 301) solutions. We were able to obtain reproducible data exhibiting anomalous rolling only in the more concentrated solutions: 1.5%, 1.25% and 1%. Anomalous rolling occurs in 0.85% solution. In the 0.75% solution, results are ambiguous, with a high degree of slipping and very little rolling, behaving like Newtonian fluids. Polyox solutions of 0.6% and less exhibit only normal rolling. We were able to enhance the angular velocity of rolling in the 1.5% Polyox solution by putting a wire on the inclined wall. This effect was not so marked in the 1.25% solution.

Figure H (previous 4.1)

Figure H. Anomalous rolling results for a 1/4-inch nylon sphere falling in aqueous Polyox in the narrow channel. (a) Normalized angular speed versus  $\theta$ . (b) Normalized angular speed versus normalized fall speed. O - 1.5% Polyox, flat surface,  $(U_0, \omega_0) = (0.434 \text{ cm/s}, 0.311 \text{ rad/s})$ ;  $\pm$  - 1.5% Polyox, wire surface,  $(U_0, \omega_0) = (0.0491 \text{ cm/s}, 0.302 \text{ rad/s})$ ;  $\infty$  - 1.25% Polyox, flat surface,  $(U_0, \omega_0) = (1.356 \text{ cm/s}, 0.388 \text{ rad/s})$ ;  $+$  - 1.25% Polyox, wire surface,  $(U_0, \omega_0) = (1.359 \text{ cm/s}, 0.278 \text{ rad/s})$ ;  $\diamond$  = 1.0% Polyox,  $(U_0, \omega_0) = (3.06 \text{ cm/s}, 0.396 \text{ rad/s})$ .

In Figure 9, we have presented measured values of  $U/U_0$  versus  $\cos \theta$  for S1. We drew the best straight line by linear curve fit without taking the data points for big rolling in which is not levitated and for hydrodynamic rolling in the vertical in which there is no normal \_\_\_\_\_ force. The data relating  $U/U_0$  and  $\omega/\omega_0$  falls close to a straight line satisfying  $\omega/\omega_0 \sim 2.37(U/U_0)$ .

In Figure 11, we have presented measured values of  $U/U_0$  versus  $\cos \theta$  for STP in two different channels and fitted the data to straight line  $\omega/\omega_0 \sim c_2(U/U_0)$ , where  $c_1 = 1.37$  for the 1/4-inch Tungsten sphere in 0.275 x 4 x 23-inch channel and  $c_1 = 1.21$  for the 1/2-inch steel ball in 0.85 x 0.85 x 18-inch square channel. Straight lines  $\omega/\omega_0 \sim c_2(U/U_0)$ , where  $c_2 =$  for Tungsten sphere and  $c_2 =$  for steel sphere are shown in Figure 12.

In Figure I, we present measured values for the normalized angular speed of spheres rolling in S1 as a function of the inclination angle  $\theta$  of the wall and the normalized fall speed  $U/U_0$ . The data appear to lie close to a straight line whose slope depends on experimental and rheological parameters in an unknown way.

Figure I (previous 4.2)

In Figure J, we have presented measured values of  $U/U_0$  and  $\omega/\omega_0$  as a function of  $\theta$ . The graph  $U/U_0$  shows that the proximity of the side wall can have a marked effect on the drag. The change of the fall and turning speed with tilt angle is greater for heavy than for high spheres and the proximity of side wall inhibits the motion for a given sphere.

In Figure J, we present measured values of  $\omega/\omega_0$  as a function of  $\theta$  and  $U/U_0$  for two different channels. The spheres fall and turn faster in the wide channels. This shows that the side walls have a marked quantitative effect on the settling and turning speed of spheres.

Figure J (previous 4.3)

Figure J. Anomalous rolling result for STP in the wide channel and in the narrow channel. (a) Normalized angular speed versus  $\theta$ . (b) Normalized angular speed versus normalized fall speed. O - 1/4-inch tungsten sphere falling in thin channel,  $(U_0, \omega_0) = (0.374 \text{ cm/s}, 0.111 \text{ rad/s})$ ,  $\pm$  = 1/2-inch steel sphere falling in wide channel,  $(U_0, \omega_0) = (0.708 \text{ cm/s}, 0.133 \text{ rad/s})$ .

##### **5. Direct numerical simulation of the settling of a circular particle in a Newtonian fluid at a vertical wall.**

The hydrodynamic mechanisms which cause circular particles to rotate and drift away from a vertical wall can be understood by direct (two-dimensional) numerical simulation, using the Navier-Stokes equations to find the fluid motion and the hydrodynamic forces which move a rigid particle according to Newton's equation of motion. A finite-element package with this capability has been presented by Hu, Joseph and Crochet [1992], and a video of this simulation together with a short paper has been given by Hu, Fortes and Joseph [1993]. Huang, Feng and Joseph [1993] applied this code to the problem of the turning couple on an elliptic particle settling in vertical channel, and they showed that there is high pressure on the front side of the ellipse at the place where the shear stress vanishes, which corresponds to a stagnation point in potential flow, which acts always to keep the long side of the body perpendicular to the fall. Feng, Hu and Joseph [1993] used this code to solve initial value

problems for circular particles in a channel and this section is an adaptation of their work to the problem at hand.

We want to understand how a heavier-than-liquid, circular particle dropped from rest in this liquid at a vertical wall will rotate and move. Referring to Feng et al. [1993] for details, we note here that in the regime of moderately low Reynolds numbers in which there is no vortex shedding, the particle will drift to channel center under the influence of side forces from both walls. The problem at hand could be posed in the semi-infinite domain on the right of the vertical wall, but is here simulated by diminishing the influence of the other wall by moving it far away, 60 sphere diameters from the active wall.

At first, when the particle is very near the wall, the fluid between the circular particle and the wall is blocked, so that the flow passes over the outside of the circular particle, turning it in the direction which we called “anomalous.” The stagnation pressure in the narrow gap induces side drift away from the wall.

In our experiments, spheres dropped from rest in glycerin would rotate drift rapidly away from the wall and after a short time reach an apparently steady state with definite angular velocity  $\omega_o$  and a fixed stand-away distance with no further side drift. In the simulation, we get a quasi-equilibrium of this type. A drift to the center of channel takes place on a much larger time scale.

In Figure A, we have drawn a cartoon of the quasi-equilibrium to help the reader to understand the fluid mechanics at work. We first put the center of the sphere to rest by a Galilean transformation. The circular particle is rotating counter-clockwise, so that the no-slip condition implies that there are no stagnation points on the surface of the circle. However, we know from our previous work that the image of the stagnation point, which we could identify for the Rankine vortex in a stream in potential flow, is the place on the front face to where the shear stress vanish, near to a dividing streamline. There is a second point on the side of the circle near the wall where the shear stress vanishes corresponding to a separation point of negative pressure. The side drift is suppressed because at the equilibrium stand-

away distance, the negative pressure has developed a sufficient magnitude to balance the stagnation pressure. Other forces enter into this balance, but only as a small change.

Figure A (same as previous 2.1)

Figure A. Cartoon of the settling of a circular particle at a wall in a coordinate system in which the center of the sphere is at rest. At the points of  $ST$  (stagnation) and  $SP$  (separation), the shear stress vanishes. Side drift is suppressed when the positive pressure at  $ST$  balances the negative pressure at  $SP$ .

## 7. Discussion and conclusions.

We have studied the motion of a sphere falling and rotating on a plane inclined with respect to gravity. The sphere is forced toward the plane by the component of its buoyant weight normal to the plane and is moved along the plane by the tangential component of the relative weight. If the normal component of the relevant weight is large enough, the sphere will make effective contact with the plane and will roll normally about the point of effective contact, as it does in air. The normal component maybe increased by tilting the plane toward the horizontal, or by increasing the weight of the sphere.

The effects of the interaction between the moving sphere and the suspending liquid introduce countervailing forces which tend to levitate the sphere and to make it rotate in the anomalous way,

opposite to what would be expected from dry rolling. The sense of rolling we have called anomalous is actually normal for the hydrodynamically-levitated case, and it could be called hydrodynamic rolling, opposite to dry rolling.

There is a marked difference in the migration of spheres dropped near to a vertical wall. In Newtonian liquids, dilute solutions and other solutions without strong viscoelastic properties, a sphere dropped in proximity of a vertical wall will be forced away from the wall by lift forces and be put into anomalous rotation by shears from the flow going around the outside of the sphere. We did an exact numerical simulation of this scenario for a circular particle falling near a wall in two dimensions and showed how the lateral motion of the particle and its equilibrium positions at the front and rear “stagnation” points near the wall where the shear stress vanishes. In a strict sense, the no slip condition in a viscous fluid is not compatible with motion of stagnation points as they appear in the theory of flows without viscosity. In the inviscid theory, stagnation points appear at dividing streamlines and a natural image of these for a viscous fluid, say with a boundary layer, are points on the boundary at which the shear stress vanishes. We find the high pressure at a point on the front face of the sedimenting circle near the “stagnation” point where the shear stress vanishes. There is also a point on the rear face where the shear stress vanishes, but without the pressure recovery and, in fact, the minimum pressure is very near to this point. The magnitude of the minimum pressure is relatively small, so that the outward drift of the particle is controlled by the component of the “stagnation pressure” on the front face pushing the particle away from the wall. We cannot carry our simulation into a semi-infinite regime, but in a channel, even with sidewalls far away, the particle will drift slowly to the center of the channel. At higher  $Re$ , after vortex shedding commences, you can see off-center \_\_\_ positions equilibrium (Feng, Hu and Joseph [1993]). It is probably that slow drift away from a wall in a semi-infinite is a permanent condition, with ever slower sidewise drifting as time goes on.

The results of the two-dimensional simulations do not give rise to the fixed stand-away distance which is observed when spheres are dropped in a Newtonian fluid near a wall. Maybe the experiments

are at fault, with channel lengths too small to see a continuous increase in the distance between the particle and the wall. Another possibility is that the theory for two dimensions is not realized in three.

The considerations of the previous paragraph do not apply to viscoelastic fluids in which particles are attracted to a wall. In this case the conclusion that there is a very small equilibrium distance between the wall and the particle is inescapable.