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Short communication

A note on the forces that move particles in a second-order fluid

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Abstract

In this note we show that the normal stresses on a solid body in plane flow of a second-order fluid are compressive and such as to turn long bodies into the stream and to cause circular particles to aggregate and chain.

Keywords: Forces; Second-order fluid

The stress in an incompressible fluid of second order is given by

$$T = -pI + \eta A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \tag{1}$$

where A_1 and A_2 are the first and second Rivlin–Ericksen tensors. The expression (1) arises at quadratic order in an expansion for slow and slowly varying motions (first proved in Ref. [1]; see Ref. [2]). Such motion can be greatly simplified in two dimensions [3] or when $\alpha_1 + \alpha_2 = (\Psi_1 + 2\Psi_2)/2 = 0$, where Ψ_1 and Ψ_2 are the first and second normal stress coefficients [4]. In either case, the velocity field is the same as that of the Stokes flow while the pressure is modified as

$$p = p_{\mathrm{N}} + \frac{\alpha_1}{\eta}\frac{\mathrm{D}p_N}{\mathrm{D}t} + \left(\frac{\alpha_2}{2} + \frac{3\alpha_1}{4}\right)A_1{:}A_1 \tag{2}$$

where p_{N} is the Stokes pressure.

For plane flows, the stress can be written as [5]

$$\begin{bmatrix} T_{xx} & T_{xy} \\ T_{xy} & T_{yy} \end{bmatrix} = -\left(p_N + \frac{\alpha_1}{\eta}\frac{\mathrm{D}p_{\mathrm{N}}}{\mathrm{D}t} + \frac{\alpha_1}{2}\Gamma\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\eta + \alpha_1 \frac{\mathrm{D}}{\mathrm{D}t}\right)\begin{bmatrix} 2a & b+c \\ b+c & 2a \end{bmatrix},$$

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$$+\alpha_1(b-c)\begin{bmatrix} -b-c & 2a \\ 2a & b+c \end{bmatrix},$$

where D/Dt is the substantial derivative, $a=\partial u/\partial x=-\partial v/\partial y$, $b=\partial u/\partial y$, $c=\partial v/\partial x$, $\alpha_1=-\Psi_1/2$ and $\Gamma=4a^2+(b+c)^2$. Now choose a generic point P on the boundary $\partial\Omega$ of the body Ω and define local coordinates (x, y) with velocity (u, v) where x is tangential and y normal to $\partial\Omega$. Since Ω is a rigid body, there is no variation of u or v along $\partial\Omega$. So $a=c=0$, $b=\dot{\gamma}$. The stress at P is

$$\begin{bmatrix} T_{xx} & T_{xy} \\ T_{xy} & T_{yy} \end{bmatrix} = -\left(p_N - \frac{\Psi_1}{2\eta}\frac{Dp_N}{Dt} - \frac{\Psi_1}{4}\dot{\gamma}^2\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\eta - \frac{\Psi_1}{4}\frac{D}{Dt}\right)\begin{bmatrix} 0 & \dot{\gamma} \\ \dot{\gamma} & 0 \end{bmatrix} - \frac{\Psi_1}{2}\dot{\gamma}^2\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \tag{4}$$

It follows that the normal component of the stress T_{yy} is given by [6]

$$T_{yy} = -p_N + \frac{\Psi_1}{2\eta}\frac{Dp_N}{Dt} - \frac{\Psi_1}{4}\dot{\gamma}^2 \tag{5}$$

The foregoing analysis works also in three dimensions when $\alpha_1=-\alpha_2(\Psi_1=-2\Psi_2)$ [4]. In either case, the total stress depends on Ψ_1, but the pressure is given by (2).

In determining the contribution of the pressure to the total normal stress in the plane case where α_2 is actually irrelevant, it is necessary to assign a value to α_2. The irrelevance of α_2 stems from the fact that in the reduction of (1) to (3), the α_2 in the pressure

$$p = p_N - \frac{\Psi_1}{2\eta}\frac{Dp_N}{Dt} + \left(\alpha_2 + \frac{3}{2}\alpha_1\right)\dot{\gamma}^2 \tag{6}$$

cancels an identical contribution in the extra stress

$$T_{yy} + p = (2\alpha_1 + \alpha_2)\dot{\gamma}^2 \tag{7}$$

The decomposition of the total normal stress into a "pressure" and extra stress is unique because of (2), but the decision to call (2) a pressure is arbitrary. Since

$$\left(\alpha_2 + \frac{3}{2}\alpha_1\right) = \hat{\beta}/2,$$

where $\hat{\beta} > 0$ is the climbing constant, and

$$2\alpha_1 + \alpha_2 = \Psi_2 < 0$$

for nearly all solutions and melts, both quadratic contributions to (6) and (7) are compressive. Moreover, in many cases $\hat{\beta}$ is large and Ψ_2 is small, so that the main compressive stresses are generated by the normal stresses in the pressure (6) [7].

The time derivative of p_N vanishes in steady flows over stationary bodies. The form of normal stresses in (6) and (7) informs intuition about how particles move and turn in a slow flow of a viscoelastic liquid; one has only to look for crowded streamlines in the Stokes flow near the body to see how the normal stresses are distributed over the body. If the particle has fore–aft symmetry, the Stokes pressure and viscous shear stress each yield a zero torque on the body; thus the normal stresses will turn the body into the stream [7,8] as in Fig. 1(a). For two identical spheres or circular cylinders settling side by side (Fig. 1(b)), strong shears occur on the outside and the resulting compressive stresses push the particles together; they then act like a long body and are turned into the stream by torques like those in Fig. 1(a). Two particles settling in tandem experience imbalanced compressive normal stresses at the bottom of the leading particle and the top of the trailing particle, causing them to chain as in Fig. 1(c). The lateral attraction of a particle to a nearby wall can be explained by a similar mechanism (Fig. 1(d)). Experimental evidence of particle–particle and particle–wall interactions has been documented in Ref. [9].

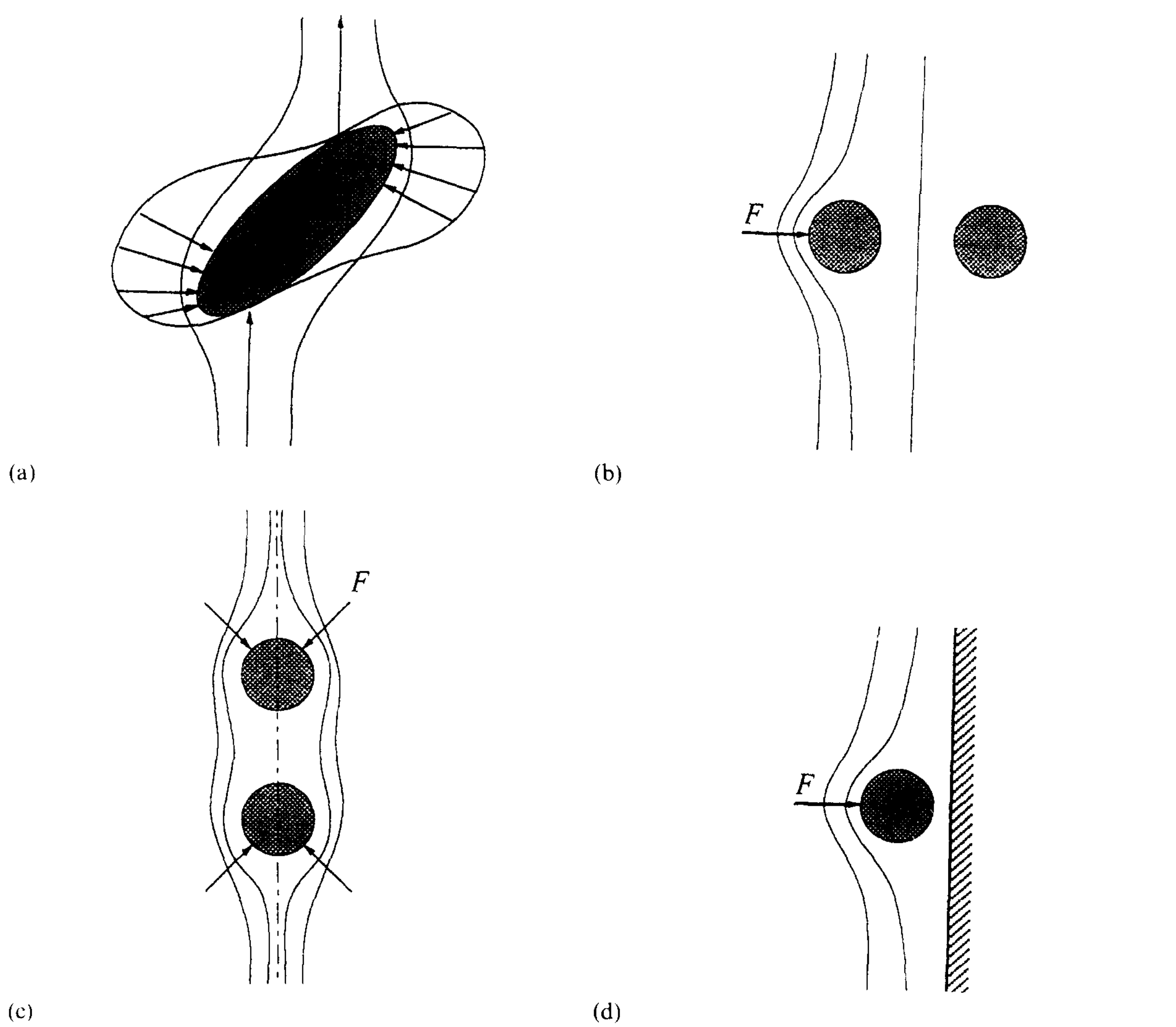

Fig. 1. Stokes flows around particles in sedimentation and the surface forces due to normal stresses. (a) Compressive surface forces on an ellipse in sedimentation. Note that the normal stress vanishes at the stagnation points of the Stokes flow. (b) Attraction between two particles settling side by side. (c) Attraction between two particles settling in tandem. (d) Wall attraction on a sedimenting particle.

The compressive stresses which are generated by the motion of particles in plane flow of a second-order fluid produce aggregation rather than dispersion; they align long bodies with the stream and produce chains of particles aligned with the stream.

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