1. Find the equation of the tangent line to the graph of

$y = \sin(x)^2$

at the point where x = π/3.

[20 marks]

$\frac{dy}{dx} = 2\sin(x)\cos(x)$ **5 marks**

At x = π/3

$\frac{dy}{dx} = 2\sin(\frac{\pi}{3})\cos(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$ **5 marks**

Solve for y when x = π/3

$y = \sin(\frac{\pi}{3})^2 = \frac{3}{4}$ **5 marks**

$y = \frac{\sqrt{3}}{2}x + b$

Solve for b

$\frac{3}{4} = \frac{\sqrt{3}}{6}\pi + b$

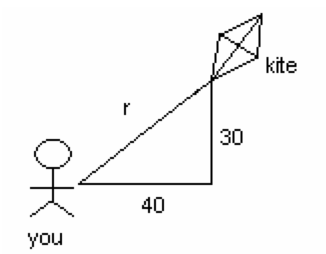
$b = \frac{3}{4} - \frac{\sqrt{3}}{6}\pi$

$y = \frac{\sqrt{3}}{2}x + \frac{3}{4} - \frac{\sqrt{3}}{6}\pi$ **5 marks**

2. You are flying a kite. At a certain time, the kite is 30 m high, 40 m horizontally away from you and moving horizontally away from you at a rate of 10 m/min. Assume that the string lies on a straight line between you and the kite at all times.

[10 marks]

a) How fast are you letting out string at that time?



Let r be the distance between you and the kite.
By the Pythagorean Theorem.

$r^2 = x^2 + y^2$ **2 marks**

Solve for r. We find r = 50.

Then by differentiating implicitly with respect to time

$2r\frac{dr}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$ **5 marks**

By substituting in values, we have:

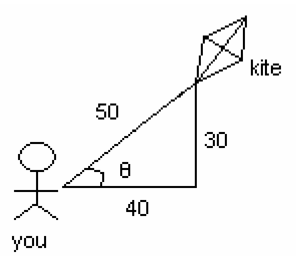
$\frac{dy}{dt} = 0 \quad \frac{dx}{dt} = 10 \quad y = 30 \quad x = 40 \quad z = 50$

$2 \cdot 50\frac{dr}{dt} = 2 \cdot 40 \cdot 10 + 2 \cdot 30 \cdot 0$

$\frac{dr}{dt} = 8 \text{ ft/min}$ **3 marks (1 mark for units)**

b) How fast is the angle between the string and the ground changing at that time?

[10 marks]



$\tan(\theta) = \frac{y}{x}$ **3 marks**

Differentiating with respect to time, we have

$\sec^2(\theta)\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$ **4 marks**

Substituting in

$\frac{dy}{dt} = 0 \quad \frac{dx}{dt} = 10 \quad y = 30 \quad x = 40 \quad \sec^2(\theta) = \frac{25}{16}$

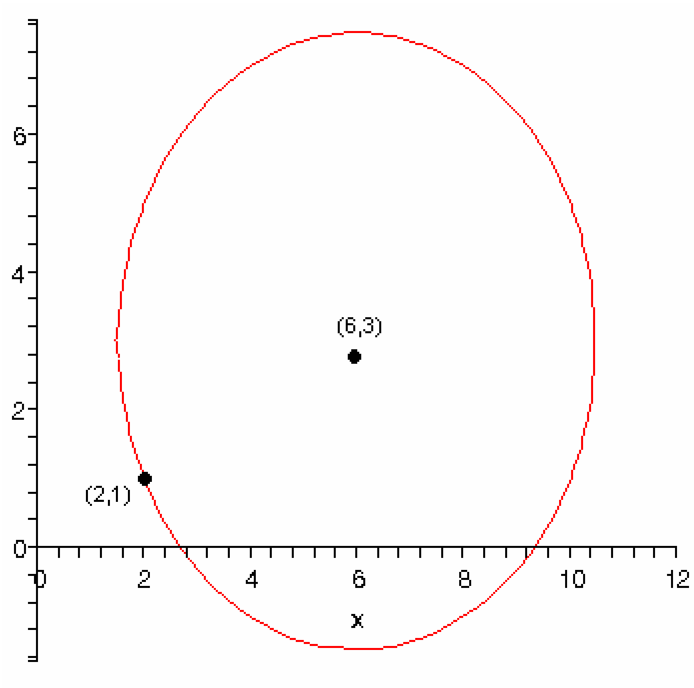
$\frac{d\theta}{dt} = \frac{-3}{25}$ radian/min **3 marks (1 mark for units)**

3.
a) Use any method to find the tangent line to the circle

$$x^2+y^2-12x-6y+25=0$$

at the point (2,1). Show your work.

[10 marks]



By implicit differentiation:

$$2x+2y\frac{dy}{dx}-12-6\frac{dy}{dx}=0$$ **5 marks**

Substituting in x=2 y=1

$$\frac{dy}{dx}=-2$$ **2 marks**

Substituting in x=2 y=1

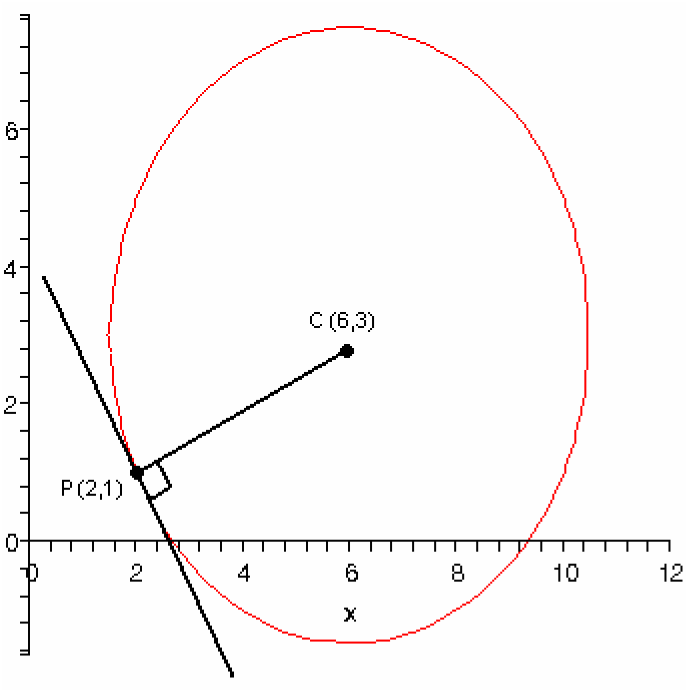
$$y=-2x+b$$

$$1=-4+b$$

$$b=5$$

$$y=-2x+5$$ **3 marks**

Alternate solution: by using Geometry



Any line segment from the center of a circle to a point P is perpendicular to the tangent line at the point P.

$$\text{Slope of CP} = \frac{\Delta y}{\Delta x} = \frac{3-1}{6-2} = \frac{1}{2}$$

The slope of a line that is perpendicular to the original line is equal to the negative-inverse slope of the original line.

$$\text{Slope of tangent line} = -\left(\frac{1}{2}\right)^{-1} = -2$$ **7 marks**

b) Using implicit differentiation find the slope of the tangent line to the circle

[10 marks]

$$x^2 + y^2 + 2x + y - 10 = 0$$

At the point (2,1).

Implicitly differentiating,

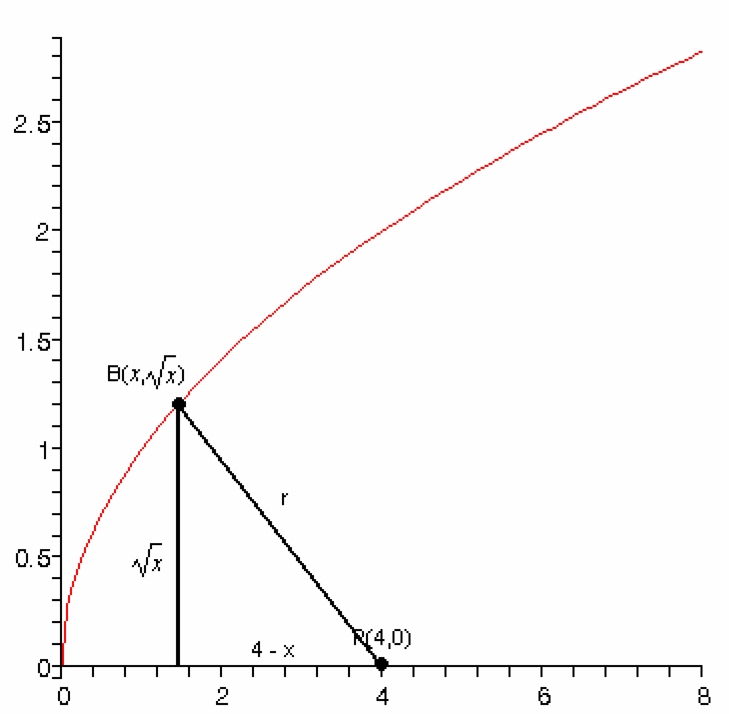
$$2x + \frac{3\,dy}{dx} + 2 = 0$$ **7 marks**

Substituting in with x=2 y=1

$$\frac{dy}{dx} = -2$$ **3 marks**

4. Find the point on the graph of $y = \sqrt{x}$ closest to the point(4,0).

[20 marks]



Let $B(x,\sqrt{x})$ be an arbitrary point on the curve $y = \sqrt{x}$.
Let r be the distance between a point on the curve and the point (4,0)

$r = \sqrt{(4-x)^2 + x} = \sqrt{16 - 7x + x^2}$ **10 marks**

$\frac{dr}{dx} = \frac{-7 + 2x}{2\sqrt{16 - 7x + x^2}}$ **5 marks**

Solving dr/dx = 0

$$\frac{dr}{dx} = \frac{-7 + 2x}{2\sqrt{16 - 7x + x^2}} = 0$$

$$-7 + 2x = 0$$

$x = \frac{7}{2}$ **5 marks**

The point on the curve $y = \sqrt{x}$ closest to the point(4,0) is $\left(\frac{7}{2}, \sqrt{\frac{7}{2}}\right)$.

5. It is desired to use Newton's Method to find the value of x at which the function

$$y = 2e^{x} + x^{2} - 4x + 1$$

has a horizontal tangent line. If you start with $x_0 = 0$ find x_1. Note that you are only required to find x_1.

[20 marks]

Note: We are trying to find when the slope is horizontal(equal to 0) so we are trying to solve when dy/dx = 0 <u>NOT</u> when y = 0, therefore, we must apply Newton's Method to dy/dx instead of y.

$$\frac{dy}{dx} = 2e^{x} + 2x - 4$$

Let h(x) be dy/dx for convenience.

$h(x) = 2e^{x} + 2x - 4$ **5 marks**

$h'(x) = 2e^{x} + 2$ **5 marks**

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ **5 marks**

$$x_1 = x_0 - \frac{h(x_0)}{h'(x_0)}$$

$x_1 = 0 - \dfrac{2e^{0} + 2\cdot 0 - 4}{2e^{0} + 2} = \dfrac{1}{2}$ **5 marks**