Set 1, Due: January 29

1. p. 47: 2-1.

2. p. 48: 2-9, 2-.10 (read the definition of gradient of a function, p. 27)

3. Let ∇ be an affine connection on an n-dimensional manifold M. Let $T : TM \times \cdots \times TM \to C^\infty(M)$ be a $(r, 0)$-type tensor. The covariant derivative ∇T is a tensor of $r+1, 0$-type defined by

$$\nabla T(X_1, ..., X_r, X) = X(T(X_1, ..., X_r)) - T(\nabla_X X_1, ..., X_r) - \cdots - T(X_1, ..., \nabla_X X_r).$$

For $X \in \Gamma(TM)$, the covariant derivative $\nabla_X T$ of T relative to X is a $(r, 0)$-type tensor given by

$$\nabla_X T(X_1, ..., X_r) = \nabla T(X_1, ..., X_r, X).$$

Let g be a Riemannian metric on M and ∇ be the Levi-Civita connection of (M, g) . Show $\nabla g = 0$.

4. Consider two parametrizations of the 2-dim torus T^2:

(a) $F_1(\alpha, \beta) = (e^{\sqrt{-1}\alpha}, e^{\sqrt{-1}\beta}) \subset \mathbb{R}^4$

(b) $F_2(\alpha, \beta) = ((2 + \cos\alpha)\cos\beta, (2 + \cos\alpha)\sin\beta, \sin\alpha) \subset \mathbb{R}^3$.

Hence T^2 is equipped with two Riemannian metrics via the pullbacks by F_1, F_2. In each case, compute $[\frac{\partial}{\partial\alpha}, \frac{\partial}{\partial\beta}]$, $\nabla_{\frac{\partial}{\partial\alpha}} \frac{\partial}{\partial\beta}$ where ∇ is the Leli-Civita connection in each case.

5. p.49: 2-15

6. p.50: 2-18