Set 4, Due: March 11, 2024

1. Let M be a Riemannian manifold with sectional curvature identically zero. Show that for every $p \in M$ the mapping $exp_p : B_r(0) \subset T_pM \to B_r(p) \subset M$ is an isometry, where $B_r(p) = exp_p(B_r(0))$ is the normal ball at p.

2. Let M be a Riemannian manifold with non-positive sectional curvature. Show that the conjugate locus $C(p) = \emptyset$ for every $p \in M$. (Hint: Assume there is a non-trivial Jacobi field J along the geodesic $\gamma : [0, a] \to M$ with $\gamma(p) = 0 = \gamma(a)$. Show $\frac{d}{dt} g(\frac{DJ}{dt}, J) \geq 0$ and conclude $g(\frac{DJ}{dt}, J) \equiv 0$. Then find a contradiction.)

3. p.185, 6-3

4. p.185, 6-4

5. p.185, 6-7