(85)

<u>Complete Manifolds</u>

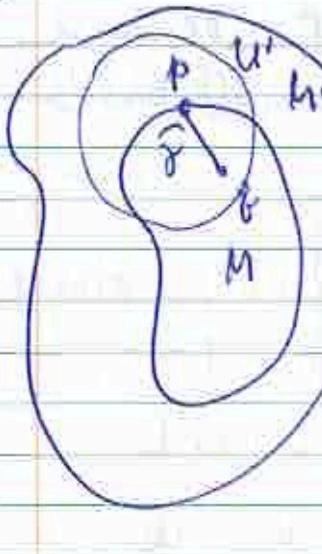
<u>Def</u> A Rie. mfd $(M.g)$ is <u>extendible</u> if $\exists$ Rie. mfd M', s.t.
M is isometric to a proper <u>open</u> subset of M'

<u>Def</u> $(M.g)$ is (geodesically) <u>complete</u> if $\forall p \in M$
$\exp_p$ is defined on ~~the~~ entire T_pM
i.e. γ defined on $\mathbb{R}$.

<u>Prop</u> If M is complete, then it is not extendible

<u>Pf</u> Suppose $M \subset M'$ iso. proper open subset
M' connected, $\partial M \subset M'$ is non empty

$p \in \partial M$, U' normal nbhd of p in M'



$q \in U' \cap M$, $\tilde{\gamma}$ geod. in M'
$\tilde{\gamma}(0) = p$, $\tilde{\gamma}(1) = q$
then $\gamma(t) = \tilde{\gamma}(1-t)$
is a geodesic in M $\gamma(0) = q$
for $|t| < \delta$ which is not
defined for some $t \le 1$,
say not for $t = 1$ $\therefore M$
cannot be complete. $\rightarrow\leftarrow$

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