

1st & 2nd variations of energy.

Def

$C: [0, a] \rightarrow M$ piecewise C^0
 A variation of C is:

$$f: (-\epsilon, \epsilon) \times [0, a] \rightarrow M$$

1) $f(0, t) = C(t)$

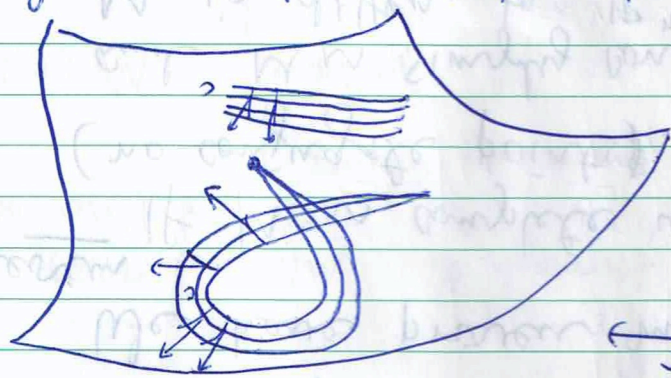
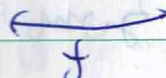
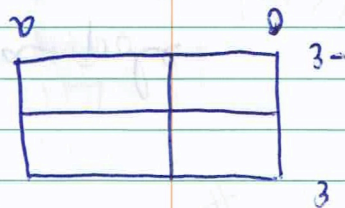
2) $f \in C^1 \mid (-\epsilon, \epsilon) \times [t_1, t_2] \subset M$

for some subdivision $t_0 = 0, t_1 < t_2 < \dots < t_n = a$.

A variation is proper if

$$f(s, 0) = C(0), \quad f(s, a) = C(a)$$

(i.e. with two end pts fixed)



$f_t(s, t) = f(s, t)$ is called a transverse

curve of the variation

and $\frac{df_t}{ds}(0, t)$ is called the variational field of f