Prop Given $c: [0,a] \to M$ and $V(t)$ along c

$\exists\, f: (-\varepsilon,\varepsilon) \times [0,a] \to M$ a variation of c

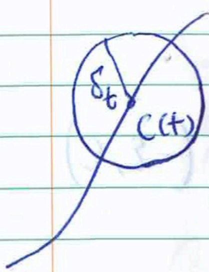
s.t. V is the v. f. i.e. $V(t) = \left.\dfrac{df(s,t)}{ds}\right|_{(0,t)}$

If $V(0) = 0 = V(a)$, f can be chosen to be proper.

pf $c([0,a])$ is compact, $\exists\, \delta > 0$

s.t. $\exp_{c(t)}$ is defined for $\forall\, v \in T_{c(t)}M$,

$|v| < \delta$, $t \in [0,a]$:



W_t : totally normal nbhd.

$\bigcup_{t\in[0,a]} W_t$ cover $c([0,a])$

$\therefore \exists$ finite cover $W_{t_1}, \dots, W_{t_n}$

$\delta = \min\{\delta_{t_1}, \dots, \delta_{t_n}\}$

$N = \max_{t\in[0,a]} |V(t)|$, $\varepsilon < \dfrac{\delta}{N}$.

Define $f(s,t) = \exp_{c(t)} sV(t)$, $s \in (-\varepsilon, \varepsilon)$, $t \in [0,a]$

$(\because |sV| < \varepsilon N < \delta)$

$\exp_{c(t)} sV(t) = \gamma(1, c(t), sV(t))$

dep'd differentiable (~~piecewise~~) on initial condition. $\therefore f$ piecewise differentiable