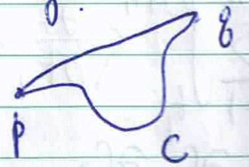
γ: minimizing geodsic

Lemma



Then $E(\gamma) \le E(C)$ and "=" $\Leftrightarrow$ C is minimizing geodesic

Pf $a E(\gamma) = (L(\gamma))^2 \le (L(c))^2 \le a E(c)$.

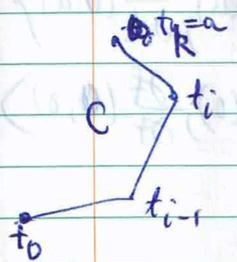
γ geod $\Rightarrow t$ prop arc-length   minimizing

"=" $\Rightarrow$ $L(C)^2 = a E(C)$ $\Rightarrow$ parameter of C is proportional to arc-length and $L(\gamma) = L(C)$ $\therefore$ C is minimizing geodsic.

RK. So E-minimizers are parametrized by a parameter which is proportional to arc-length.

Prop (1st Vari. formula of E).

$$\frac{1}{2} E'(0) = -\int_0^a \left\langle V(t), \frac{D}{dt}\frac{dc}{dt} \right\rangle - \sum_{i=1}^{k} \left\langle V(t_i), \frac{dc}{dt}(t_i^+) - \frac{dc}{dt}(t_i^-) \right\rangle - \left\langle V(0), \frac{dc}{dt}(0) \right\rangle + \left\langle V(a), \frac{dc}{dt}(a) \right\rangle$$



where V is the vari. field of f.