

$$E(s) = \int_a^b \left\langle \frac{\partial^2}{\partial t^2}, \frac{\partial^2}{\partial t^2} \right\rangle = \sum_{j=1}^n \int_{t_j}^{t_{j+1}} \left\langle \frac{\partial^2}{\partial t^2}, \frac{\partial^2}{\partial t^2} \right\rangle$$

$$= \int_{t_1}^{t_2} \left\langle \frac{\partial^2}{\partial t^2}, \frac{\partial^2}{\partial t^2} \right\rangle = 2 \int_{t_1}^{t_2} \left\langle \frac{\partial}{\partial t}, \frac{\partial}{\partial t} \right\rangle$$

$$= 2 \int_{t_1}^{t_2} \left\langle \frac{\partial}{\partial t}, \frac{\partial}{\partial t} \right\rangle = 2 \int_{t_1}^{t_2} \left\langle \frac{\partial}{\partial t}, \frac{\partial}{\partial t} \right\rangle$$

$$= 2 \int_{t_1}^{t_2} \left\langle \frac{\partial}{\partial t}, \frac{\partial}{\partial t} \right\rangle - 2 \int_{t_1}^{t_2} \left\langle \frac{\partial}{\partial t}, \frac{\partial}{\partial t} \right\rangle$$

$$= 2 \left\langle \frac{\partial}{\partial t}, \frac{\partial}{\partial t} \right\rangle \Big|_{t_1}^{t_2} - 2 \int_{t_1}^{t_2} \left\langle \frac{\partial}{\partial t}, \frac{\partial}{\partial t} \right\rangle$$

$$\therefore \frac{1}{2} E'(s) = - \int_a^b \left\langle \frac{\partial^2}{\partial t^2}, \frac{\partial^2}{\partial t^2} \right\rangle + \sum_{j=1}^n \left\langle \frac{\partial^2}{\partial t^2}, \frac{\partial^2}{\partial t^2} \right\rangle \Big|_{t_j}^{t_{j+1}}$$

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RR. ① If  $e$  is  $C^2$  then

$$\frac{1}{2} E'(s) = - \int_a^b \left\langle V, \frac{\partial^2}{\partial t^2} \right\rangle + \left\langle V(a), \frac{\partial^2}{\partial t^2}(a) \right\rangle - \left\langle V(b), \frac{\partial^2}{\partial t^2}(b) \right\rangle$$

If further,  $e$  is closed

$$\frac{1}{2} E'(s) = - \int_a^b \left\langle V, \frac{\partial^2}{\partial t^2} \right\rangle$$