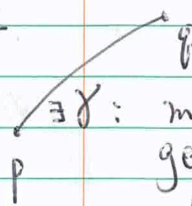
Theorem of Bonnet-Myers

M^n: complete, $Ric_p(v) \geq \frac{1}{r^2} > 0$, $\forall p, v \in T_pM$

Then M is compact and the diameter $\leq \pi r$.

<u>Pf</u>



M complete

$\Downarrow$

$\exists \gamma$: minimizing geodesic. $\gamma(0) = p$, $\gamma(1) = q$

We only need to show: $l(\gamma) \leq \pi r$

(complete + bounded $\Rightarrow$ compact)

If $l(\gamma) > \pi r$. let $e_1(t), \cdots, e_{n-1}(t)$ be parallel o.n. fields along γ and $e_i(t) \perp \gamma'(t)$.

Set $e_n(t) = \gamma'(t)/l$

$$V_j(t) = (\sin \pi t)\, e_j(t), \quad j = 1, \cdots, n-1$$

$$\therefore V_j(0) = 0 = V_j(1)$$

So V_j generate proper variations of γ with energy $E_j(t)$.

By 2^{nd} variation formula of E,