Def. Distance function on (M, g)

$$d(p,q) = \inf_{\gamma} l(\gamma), \quad \gamma \text{ from } p \text{ to } q \text{ piecewise } C^\infty$$

Prop. (M, d) is a metric space:

1) $d(p,r) \leq d(p,q) + d(q,r)$
2) $d(p,q) = d(q,p)$
3) $d(p,q) \geq 0$, "$= 0$" iff $p = q$

Pf 1), 2). from inf

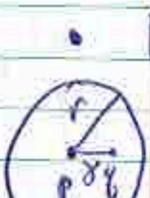
3). If $p \neq q$.

$r > 0$. $d(p,q) = 0 \Rightarrow \exists\, C$ s.t. $C(0) = p$, $C(1) = q$.

normal ball $B_r(p)$. $l(C) < r$

But $l(C \cap B_r(p)) \geq r$ because radial geod. in normal ball minimize length. $\rightarrow\leftarrow$ #

- If $0 < r \ll 1$, then $d(p,q) = l(\gamma)$, $\gamma(0) = p$, $\gamma(1) = q$



$\therefore$ normal ball of radius r

= metric ball of radius r

$\therefore$ Topology on M from d is the same one from g.