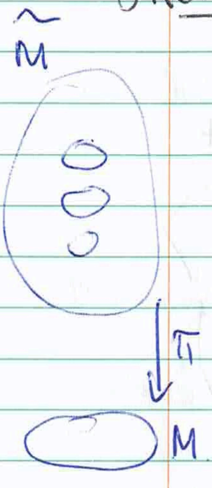
Sketch of Proof.



$\pi : \tilde{M} \to M$. $\tilde{M}$ carries the covering metric (π is a local isometry w.r.t. this metric).

$\therefore$ $Ric(\tilde{M}) \geq \delta$ $\therefore$ $\tilde{M}$ compact

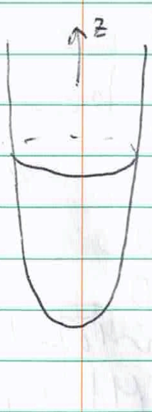
$\therefore$ # of sheets of the covering is finite, this number is equal to the # of elements in $\pi_1(M)$.

Corollary M: complete, $K \geq \frac{1}{r^2} > 0$.

Then M is compact, $\operatorname{diam}(M) \leq \pi r$
$\pi_1(M)$ is finite.

Remark. 1) Consider the paraboloid in $\mathbb{R}^3$:

$\{ (x, y, z) : z = x^2 + y^2 \}$



$K > 0$, complete but not compact

2) If K does not approach to 0 too fast, then the result still holds
(Calabi, 1967, Duke J. Math)