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- $\alpha(X,Y) = \bar{\nabla}_{\bar{X}}\bar{Y} - \nabla_X Y \quad \in TM^\perp$

is indep't of extension:

$$(\bar{\nabla}_{\bar{X}}\bar{Y} - \nabla_X Y) - (\bar{\nabla}_{\bar{X}_1}\bar{Y} - \nabla_X Y) = \bar{\nabla}_{\bar{X}-\bar{X}_1}\bar{Y} = 0 \text{ on } M$$

$$(\bar{\nabla}_{\bar{X}}\bar{Y} - \nabla_X Y) - (\bar{\nabla}_{\bar{X}}\bar{Y}_1 - \nabla_X Y) = \bar{\nabla}_X(\bar{Y} - \bar{Y}_1) = 0$$

- α is bilnear
- α is symmetric

$$\begin{aligned}\alpha(X,Y) &= \bar{\nabla}_{\bar{X}}\bar{Y} - \nabla_X Y \quad \text{on } M \\ &= \bar{\nabla}_{\bar{Y}}\bar{X} + [\bar{X},\bar{Y}] - \nabla_Y X - [X,Y] \\ &= \alpha(Y,X)\end{aligned}$$

<u>Def</u>. $\mathrm{II}_\nu(x) := \langle \alpha(\cdot,\cdot), \nu \rangle, \quad \nu \in TM^\perp$

is a map from $TM \times TM \to \mathbb{R}$
is called the 2^{nd} fundamental form of M at x along ν

α: 2^{nd} fund. form.

The shape operator $A_\nu : TM \to TM$ is defined by

$$g(A_\nu(X), Y) = g(\alpha(X,Y), \nu), \ \forall Y \in TM$$