

$$g(A_p(X)Y) = g(X(X)Y)$$

$$= g(\nabla^X Y - \nabla^X X)$$

$$= g(\nabla^X Y)$$

$$= -g(Y, \nabla^X X)$$

$$A_Y = -(\nabla^X X)^T$$

Ex hypersurface:  $M^n \hookrightarrow M^{n+1}$

$$\therefore g(A_Y(X), Y) = g(A_Y(Y), X)$$

i.e.  $A_Y$  is symmetric,

$\exists$  o.n.  $e_1, \dots, e_n \in TM$

$$A_Y(e_i) = \lambda_i e_i$$

Assume  $\{e_1, \dots, e_n\}$ ,  $\{\underline{e}_1, \dots, \underline{e}_n\}$  define orientations on  $M, \underline{M}$

$\lambda_i$ : principle curvatures

$e_i$ : principle directions

$\frac{1}{2} \Delta X$ : mean curvature of  $f$

$\lambda_1 \dots \lambda_n$ : Gauss-Kronecker curvature