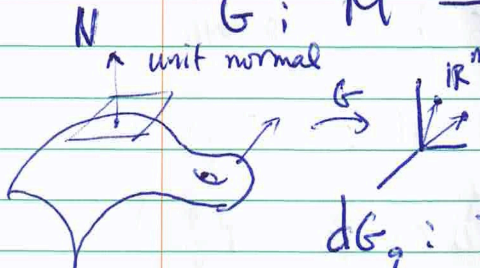
When $\bar{M} = \mathbb{R}^{n+1}$, Gauss map

$$G : M^n \longrightarrow S^n$$



$T_qM \mathbin{/\!/} T_{G(q)}S^n$

$$dG_q : T_qM \longrightarrow T_qM$$

$$dG_q(X) = \frac{d}{dt} N \circ c(t) \Big|_{t=0}$$

$$= \bar{\nabla}_X N = (\bar{\nabla}_X N)^T = -A_N(X)$$

$$c : (-\varepsilon, \varepsilon) \rightarrow M, \quad c(0) = q, \quad c'(0) = X$$

$$0 = \bar{\nabla}_X \underbrace{g(N,N)}_{=1} = 2g(N, \bar{\nabla}_X N)$$

$$\therefore \quad A_N = -dG$$

Theorem (Gauss' Equation)

$$\bar{R}(W,Z,X,Y) = R(W,Z,X,Y) + g(\alpha(X,Z), \alpha(Y,W)) - g(\alpha(Y,Z), \alpha(X,W))$$

$\forall X, Y, Z, W \in TM$.