(87)

- distance function $d(p,\cdot): M \to \mathbb{R}$ is continuous

<u>Theorem</u> (Hopf-Rinow) (M, g)

a) M is geod. complete

$\Leftrightarrow$ b) M is metrically complete

$\Leftrightarrow$ c) closed & bound sets of M are compact

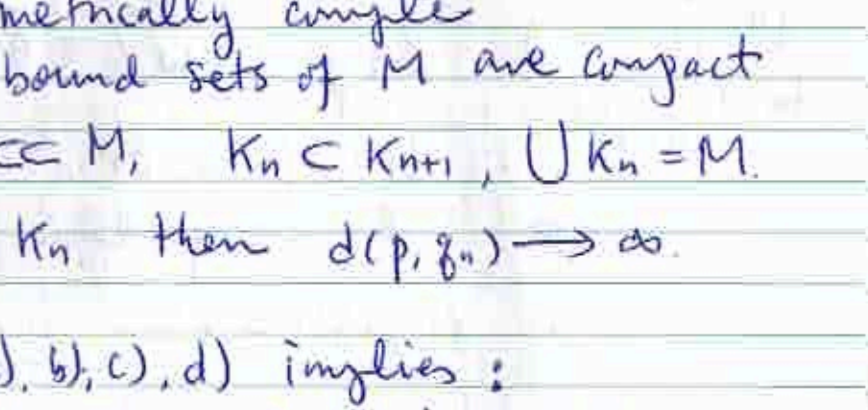
general topology $\Leftrightarrow$ d) $\exists\ K_n \subset\subset M$, $K_n \subset K_{n+1}$, $\bigcup K_n = M$ (if M non cpt) s.t. if $q_n \notin K_n$ then $d(p, q_n) \longrightarrow \infty$.

Moreover,
any of a), b), c), d) implies:

(*) $\forall p, q \in M$, $\exists$ geod. γ joining p, q
$\ell(\gamma) = d(p,q)$.

<u>Pf</u> a) $\Rightarrow$ (*):

$d(p,q) = r$



Consider
$A = \{s \in [0,r] : d(q, \gamma(s)) = r - s\}$. $0 \in A$, A closed

let $s_0 \in A$. If $s_0 < r$