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∇^\perp is a connection:

$$\begin{aligned} 1) \nabla_X^\perp(f\eta) &= \bar{\nabla}_X^\perp(f\eta) + A_\eta(fX) \\ &= f \bar{\nabla}_X^\perp \eta + f A_\eta(X) + Xf \cdot \eta \\ &= f \nabla_X^\perp \eta + Xf \cdot \eta \end{aligned}$$

2) linear in X.

Normal curvature:

$$R^\perp(X, Y)\eta = \nabla_Y^\perp \nabla_X^\perp \eta - \nabla_X^\perp \nabla_Y^\perp \eta + \nabla_{[X, Y]}^\perp \eta$$

Prop. Ricci Equation

$$\bar{R}(X, Y, \eta, \xi) - R^\perp(X, Y, \eta, \xi) = \bar{g}([A_\eta, A_\xi]X, Y)$$

$$\begin{aligned} \text{PF. } \bar{R}(X, Y)\eta &= \bar{\nabla}_Y \bar{\nabla}_X \eta - \bar{\nabla}_X \bar{\nabla}_Y \eta + \bar{\nabla}_{[X, Y]} \eta \\ &= \bar{\nabla}_Y (\nabla_X^\perp \eta - A_\eta(X)) - \bar{\nabla}_X (\nabla_Y^\perp \eta - A_\eta(Y)) \\ &\quad + \nabla_{[X, Y]}^\perp \eta - A_\eta([X, Y]) \end{aligned}$$

$$\begin{aligned} &= \bar{\nabla}_Y \nabla_X^\perp \eta - A_{\nabla_X^\perp \eta}(Y) - \bar{\nabla}_Y A_\eta(X) \\ &\quad - \nabla_X^\perp \nabla_Y^\perp \eta + A_{\nabla_Y^\perp \eta}(X) + \bar{\nabla}_X A_\eta(Y) \end{aligned}$$