$\nabla^\perp$ is a connection:

1) $\nabla^\perp_X(f\eta) = \bar\nabla_X(f\eta) + A_\eta(fX)$

$$= f\,\bar\nabla_X\eta + f A_\eta(X) + Xf\cdot\eta$$

$$= f\,\nabla^\perp_X\eta + Xf\cdot\eta$$

2) linear in X.

Normal curvature:

$$R^\perp(X,Y)\eta = \nabla^\perp_Y\nabla^\perp_X\eta - \nabla^\perp_X\nabla^\perp_Y\eta + \nabla^\perp_{[X,Y]}\eta$$

__Prop.__ Ricci Equation

$$\bar R(X,Y,\eta,\zeta) - R^\perp(X,Y,\eta,\zeta) = \bar g([A_\eta, A_\zeta]X, Y)$$

__Pf.__ $\bar R(X,Y)\eta = \bar\nabla_Y\bar\nabla_X\eta - \bar\nabla_X\bar\nabla_Y\eta + \bar\nabla_{[X,Y]}\eta$

$$= \bar\nabla_Y(\nabla^\perp_X\eta - A_\eta(X)) - \bar\nabla_X(\nabla^\perp_Y\eta - A_\eta(Y))$$

$$+ \nabla^\perp_{[X,Y]}\eta - A_\eta[X,Y]$$

$$= \nabla^\perp_Y\nabla^\perp_X\eta - A_{\nabla^\perp_X\eta}(Y) - \bar\nabla_Y A_\eta(X)$$

$$- \nabla^\perp_X\nabla^\perp_Y\eta + A_{\nabla^\perp_Y\eta}(X) + \bar\nabla_X A_\eta(Y)$$