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Prop Codazzi Equation

$$(\bar{\nabla}_Y \alpha)(X, z, \eta) - (\bar{\nabla}_X \alpha)(Y, z, \eta) = \bar{R}(X, Y, z, \eta)$$

Remark: $\alpha: TM \times TM \times TM^\perp \rightarrow \mathbb{R}$

$$\text{by } \alpha(X, Y, \eta) = g(\alpha(X, Y), \eta)$$

$$\begin{aligned} (\bar{\nabla}_X \alpha)(Y, z, \eta) &= X \alpha(Y, z, \eta) - \alpha(\nabla_X Y, z, \eta) \\ &\quad - \alpha(Y, \nabla_X z, \eta) - \alpha(Y, z, \nabla_X^\perp \eta) \end{aligned}$$

Pf of Codazzi Eq:

$$\begin{aligned} (\bar{\nabla}_X \alpha)(Y, z, \eta) &= X \bar{g}(\alpha(Y, z), \eta) - \bar{g}(\alpha(\nabla_X Y, z), \eta) \\ &\quad - \bar{g}(\alpha(Y, \nabla_X z), \eta) - \bar{g}(\alpha(Y, z), \nabla_X^\perp \eta) \\ &= \bar{g}(\nabla_X^\perp \alpha(Y, z), \eta) - \bar{g}(\alpha(\nabla_X Y, z), \eta) \\ &\quad - \bar{g}(\alpha(Y, \nabla_X z), \eta) \end{aligned}$$

$$\begin{aligned} (\bar{\nabla}_Y \alpha)(X, z, \eta) &= \bar{g}(\nabla_Y^\perp \alpha(X, z), \eta) - \bar{g}(\alpha(\nabla_Y X, z), \eta) \\ &\quad - \bar{g}(\alpha(X, \nabla_Y z), \eta) \end{aligned}$$

$$\begin{aligned} \therefore (\bar{\nabla}_Y \alpha)(X, z, \eta) - (\bar{\nabla}_X \alpha)(Y, z, \eta) \\ = \bar{g}(\nabla_Y^\perp \alpha(X, z), \eta) - \bar{g}(\nabla_X^\perp \alpha(Y, z), \eta) \end{aligned}$$