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If  $M^n \hookrightarrow \bar{M}^{n+1}$ ,  $\bar{M}$  has const. sect. curv.

Note  $\nabla_x \eta \cdot \eta = 0, \therefore \nabla_x \eta \perp \eta$

$$\therefore \nabla_x \eta = 0$$

$$\begin{aligned} \therefore (\bar{\nabla}_x \alpha)(Y, z, \eta) &= X \bar{g}(A_\eta Y, z) - \bar{g}(A_\eta \nabla_x Y, z) - \bar{g}(A_\eta Y, \nabla_x z) \\ &= \bar{g}(\nabla_x (A_\eta Y), z) - \bar{g}(A_\eta \nabla_x Y, z) \end{aligned}$$

$\therefore$  Codazzi eq:

$$\nabla_x (A_\eta Y) - \nabla_Y (A_\eta X) = A_\eta [X, Y].$$

$$\therefore (\bar{\nabla}_x \alpha)(Y, z, \eta) = (\bar{\nabla}_Y \alpha)(X, z, \eta)$$

$$\begin{aligned} \parallel & \qquad \qquad \qquad \parallel \\ \bar{g}(\nabla_x (A_\eta Y), z) - \bar{g}(A_\eta \nabla_x Y, z) &= \bar{g}(\nabla_Y (A_\eta X), z) - \bar{g}(A_\eta \nabla_Y X, z) \end{aligned}$$