(84)

If $M^n \hookrightarrow \bar{M}^{n+1}$, $\bar{M}$ has const. sect. curv.

note $\bar{\nabla}_X \eta \cdot \eta = 0$, $\therefore \bar{\nabla}_X \eta \perp \eta$

$\therefore \nabla^{\perp}_X \eta = 0$

$\therefore (\bar{\nabla}_X \alpha)(Y, Z, \eta) = X\,\bar{g}(A_\eta Y, Z) - \bar{g}(A_\eta \nabla_X Y, Z) - \bar{g}(A_\eta Y, \nabla_X Z)$

$= \bar{g}(\nabla_X (A_\eta Y), Z) - \bar{g}(A_\eta \nabla_X Y, Z)$

$\therefore$ Codazzi eq:

$\nabla_X (A_\eta Y) - \nabla_Y (A_\eta X) = A_\eta [X, Y]$.

$\Uparrow$

$\therefore (\bar{\nabla}_X \alpha)(Y, Z, \eta) = (\bar{\nabla}_Y \alpha)(X, Z, \eta)$

$\bar{g}(\nabla_X (A_\eta Y), Z) - \bar{g}(A_\eta \nabla_X Y, Z) \qquad \bar{g}(\nabla_Y (A_\eta X), Z) - \bar{g}(A_\eta \nabla_Y X, Z)$