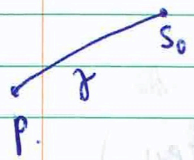
(89)

(1) Cauchy $\Rightarrow \{p_n\}$ converges.

b) $\Rightarrow$ a) If M is not geod. complete.



$\exists$ geod. γ, defined for $s < s_0$ but not for s_0 $\quad |\dot\gamma| = 1$

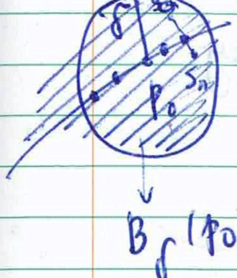
let $s_n \to s_0$, $s_n < s_0$.

$\forall \varepsilon > 0$, $\exists n_0$, $\forall m, n > n_0$,

$$d(\gamma(s_n), \gamma(s_m)) \leq |s_n - s_m| < \varepsilon$$

$\therefore$ $\gamma(s_n)$ is Cauchy in (M, d)

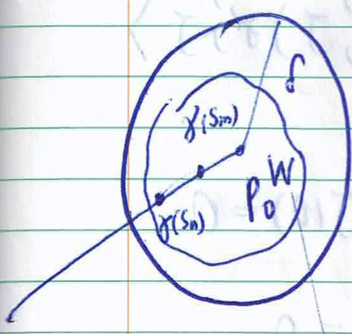
$\because$ (M, d) complete, $\exists p_0 \in M$, $\gamma(s_n) \to p_0$.



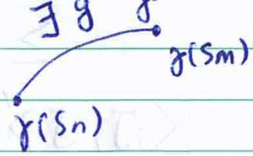
$\forall n, m > n_1$, $\quad |s_n - s_m| < \delta$

and $\quad \gamma(s_n) \in B_\delta(p_0)$

$\exp_{\gamma(s_n)}$ is diffeomorphic on $B_\delta(0) \supseteq W$



$\exists g$ geod.



g extends
$g = \gamma$ whenever γ defined
$\therefore \gamma$ extends.

W: totally normal nbhd, i.e. $\forall$ two pts, $\exists!$ minimizing geodesic connecting them

<u>Cor</u> (1) compact M $\Rightarrow$ complete

(2) A closed submfd of a complete mfd is complete in induced metric.