Pf of the Hadamard Theorem:

M complete $\Rightarrow$ $\exp_p$ is defined for all p and surjective, is locally diffeo.

Use $\exp_p$ to pull back the metric g on M to T_pM, w.r.t. which $\exp_p$ is a local isometry. $\therefore |\exp_{p*} v| = |v|$ This metric is complete because geodesics of T_pM through p are straight lines. (Hopf-Rinow) $\exp_p$ is a covering map + M is simply connected $\Rightarrow$ $\exp_p$ is diffeomorphic.

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We have proven more:

Theorem

If M is complete with a pole (no conjugate points along any geodesic) and M is simply connected, then M is diffeo. to $\mathbb{R}^n$.