REAL ANALYSIS 18.100C: WHAT'S ON THE FINAL?

Here is a list of topics which I think are important and you should definitely be familiar with.

> Chapter 4: Continuous functions, relation between continuous functions and open /closed/compact/connected sets, uniform continuity.
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> Chapter 5: Definition of differentiation, Mean Value theorems: 5.8, 5.9, 5.10, 5.11, Taylor's theorem.
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> Chapter 6: Definition of Riemann-Stieltjes integration, Theorem 6.6, Theorem 6.8, basic properties of the Riemann-Stieljes integral (Theorem 6.12), Theorems 6.15, 6.16, 6.17, Fundamental Theorem of Calculus: Theorems 6.20, 6.21, 6.22.
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> Chapter 7: Pointwise vs uniform convergence: examples illustrating the difference between the two notions, uniform convergence and continuity: Theorems 7.12 and 7.15, uniform convergence and integration/differentiation: Theorems 7.16, 7.17, Definitions of Equicontinuity, Uniform boundedness and pointwise boundedness, The Arzela-Ascolii theorem, The Stone-Weirstrass theorem.

Here are some practice questions:

Determine whether the following statements are true or false.

(1) If $f \in \mathcal{R}(\alpha)$ on $[a, b]$, then $f \in \mathcal{R}(\alpha)$ on any $[c, d] \subseteq [a, b]$.
(2) Suppose that $f, \alpha : [a, b] \longrightarrow (0, 75]$ are discontinuous at the same point $x \in [a, b]$, then $f \notin \mathcal{R}(\alpha)$.
(3) The uniform closure $\overline{\mathcal{F}}$ of an equicontinuous set $\mathcal{F} \subseteq C([0, 1])$ is equicontinuous.
(4) A finite set of continuous functions is equicontinuous.
(5) The set $\{x^n\} \subseteq C^0([0, 1])$ is equicontinuous.
(6) If f^2 is integrable, then f is integrable.
(7) If f^3 is integrable, then f is integrable.
(8) If $f \in \mathcal{R}$ with $\int f = 0$ and $f \geq 0$, then $f = 0$.
(9) In a complete metric space, closed and bounded sets are compact.
(10) The pointwise limit of a sequence of continuous functions on a compact set is continuous.
(11) The preimage of a compact set by a continuous function is compact.
(12) Suppose that $f_n \to f$ uniformly and that f_n is differentiable. Then, $f_n' \to f'$ pointwise.
(13) The set $\{f_n\}$ is equicontinuous if $f_n \to f$ uniformly.
(14) The pointwise limit of a sequence of polynomials, $P_n : [0, 1] \to \mathbb{R}$, can be discontinuous.
(15) The uniform limit of a sequence of polynomials, $P_n : [0, 1] \to \mathbb{R}$, can be discontinuous.
(16) The uniform limit of a sequence of polynomials, $P_n : [0, 1] \to \mathbb{R}$, can be non-differentiable everywhere.